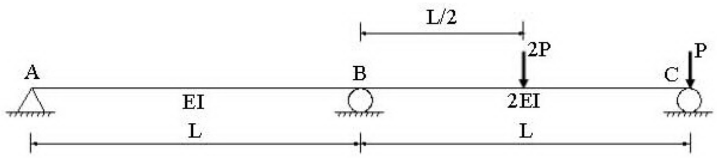
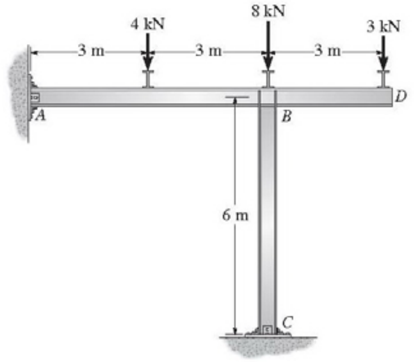
國立臺灣海洋大學河海工程學系 2025 結構學 3A 班第三次大考參考解答

日期：2026 年 01 月 09 日　　姓名：______________　　學號：______________

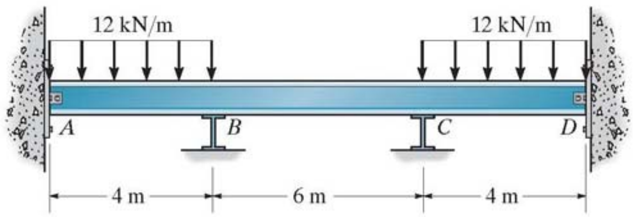
1. 請以傾角變位法(slope-deflection method)分析下圖梁結構，試求 A、B 及 C 點之反力並繪製剪力圖與彎矩圖，假設所有桿件之 EI = constant 。 (20 分)



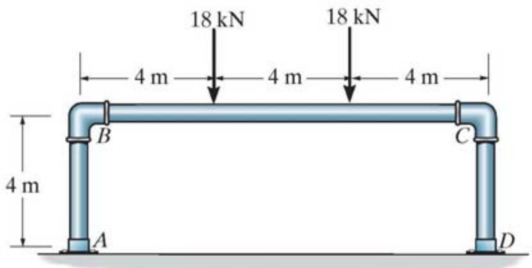
2. 如下圖，假設支承 A 與 C 點及接點 B 為固定連接，且 EI 為定值，試以傾角變位法(slope-deflection method)求出剛架各構件末端的彎矩。 (20 分)



3. 如下圖，若 EI 為定值，假設 B 與 C 點為滾支承，A 與 D 點為樞支承，試以彎矩分配法求出 B 及 C 處的彎矩，並繪出該梁的彎矩圖。(20 分)



4. 如下圖，剛架由固定連接的管子製成，設 EI 為定值，若承受如圖所示之載重，試以彎矩分配法求出在各接點和支承處產生的彎矩。(20 分)

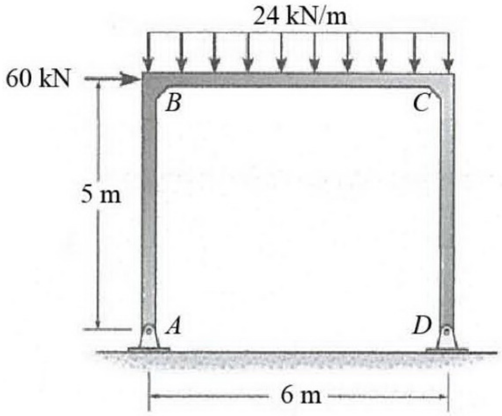


5. 如下圖，若 EI 為定值

(1) 限以傾角變位法求各端點彎矩與構架的側位移。(16 分)

(2) 限以彎矩分配法求各端點彎矩與構架的側位移。(16 分)。

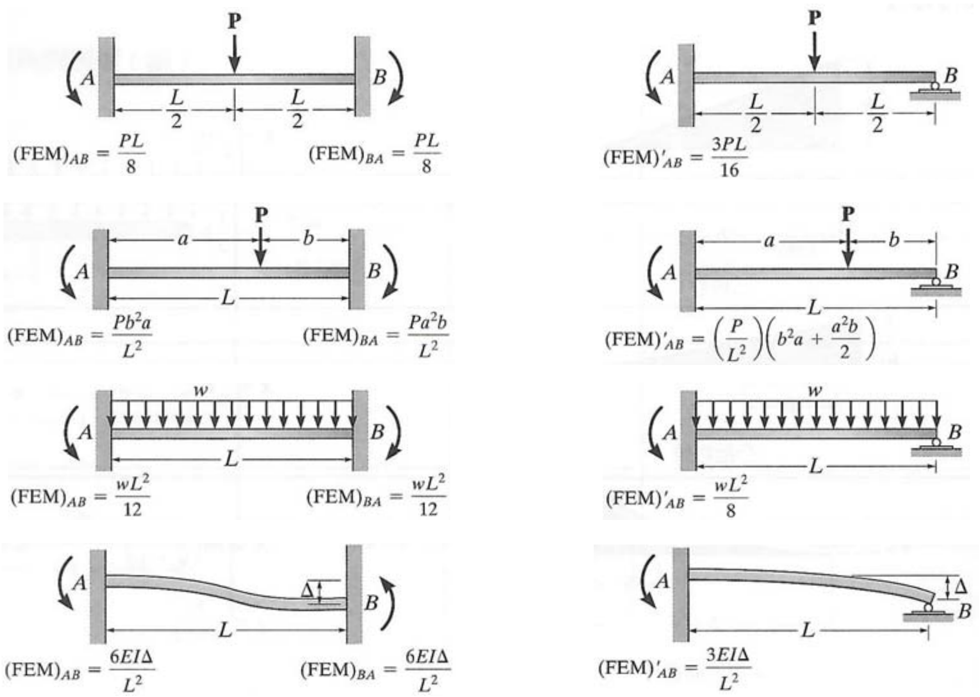
(3) 請畫出此構架的剪力圖與彎矩圖。(8 分)



6. 請問上完這學期結構學後有何心得與建議? (10%)

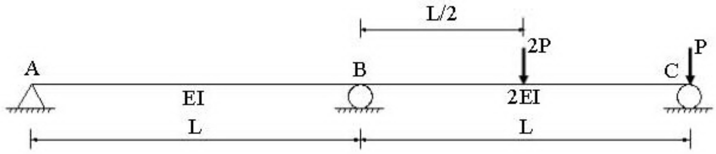
參考公式:

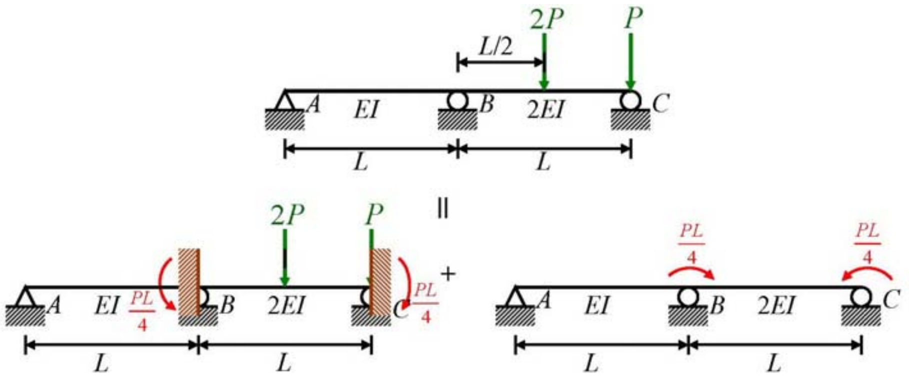
固端彎矩



參考解答:

1. 請以傾角變位法(slope-deflection method)分析下圖梁結構，試求 A、B 及 C 點之反力。假設梁之 $EI=\text{constant}$ 。 (20 分)



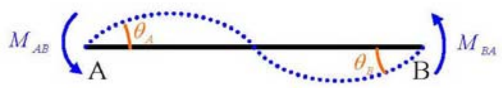


$$(\text{FEM})_{BC}=\frac{PL}{4}，(\text{FEM})_{CB}=-\frac{PL}{4}$$ (正表示逆時鐘方向，負表示順時鐘方向)

AB 段 ($M_{AB}=0$)

$$M_{AB}=\frac{4EI}{L}\theta_A+\frac{2EI}{L}\theta_B \quad \Rightarrow \theta_A=-\frac{1}{2}\theta_B$$

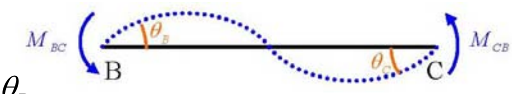
$$M_{BA}=\frac{2EI}{L}\theta_A+\frac{4EI}{L}\theta_B=\frac{3EI}{L}\theta_B$$



BC 段 ($M_{CB}=\frac{PL}{4}$)

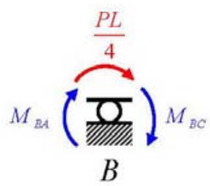
$$M_{BC}=\frac{4(2EI)}{L}\theta_B+\frac{2(2EI)}{L}\theta_C=\frac{8EI}{L}\theta_B+\frac{4EI}{L}\theta_C$$

$$M_{CB}=\frac{2(2EI)}{L}\theta_B+\frac{4(2EI)}{L}\theta_C \quad \Rightarrow \theta_C=\frac{PL^2}{32EI}-\frac{1}{2}\theta_B$$



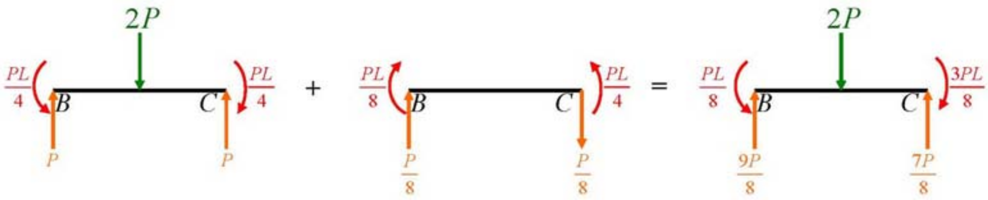
代回 M_{BC} 可得 $M_{BC}=\frac{6EI}{L}\theta_B+\frac{PL}{8}$

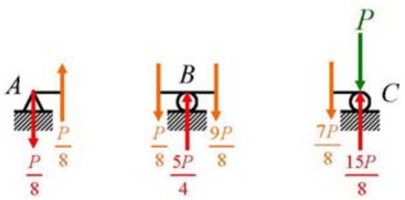
取 B 節點彎矩平衡: $M_{BA}+M_{BC}+\frac{PL}{4}=0 \quad \Rightarrow \theta_B=-\frac{PL^2}{24EI}$



$\therefore \quad \theta_A=-\frac{1}{2}\theta_B=\frac{PL^2}{48EI}， \quad \theta_C=\frac{PL^2}{32EI}-\frac{1}{2}\theta_B=\frac{5PL^2}{96EI}$

故可得 $M_{BA}=\frac{3EI}{L}\theta_B=-\frac{PL}{8}， \quad M_{BC}=\frac{6EI}{L}\theta_B+\frac{PL}{8}=-\frac{PL}{8}$



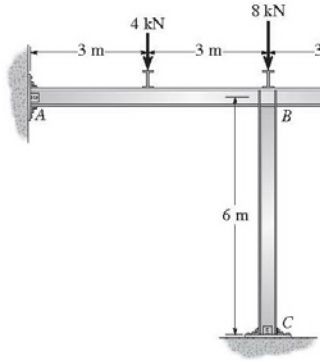


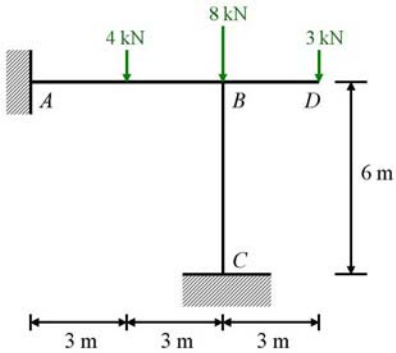
A 點反力 $\dfrac{P}{8}$(向下)

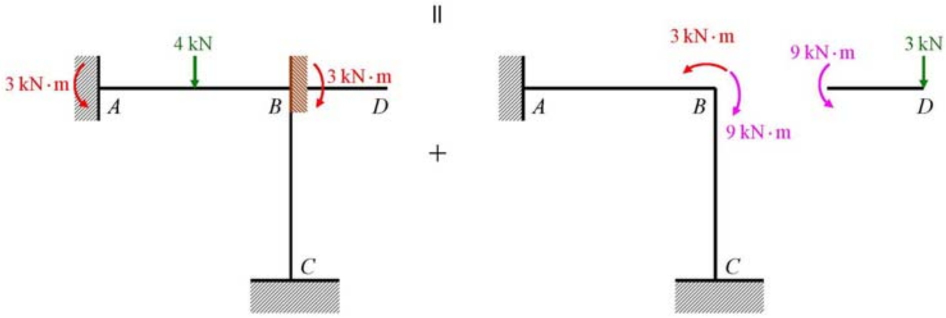
B 點反力 $\dfrac{5P}{4}$(向上)

C 點反力 $\dfrac{15P}{8}$(向上)

2. 如下圖，假設支承 A 與 C 點及接點 B 為固定連接，且 EI 為定值，試以傾角變位法(slope-deflection method)求出剛架各構件末端的彎矩。 (20 分)



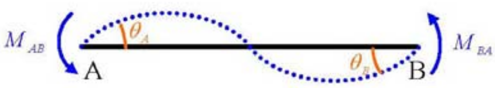




由圖可知 A 與 C 端皆為固定端，即 $\theta_A = 0$ 與 $\theta_C = 0$

AB 段（$\theta_A = 0$）

$$M_{AB} = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B \quad \Rightarrow M_{AB} = \frac{EI}{3}\theta_B$$

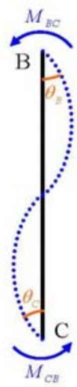


$$M_{BA} = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B = \frac{3EI}{L}\theta_B \quad \Rightarrow M_{BA} = \frac{2EI}{3}\theta_B$$

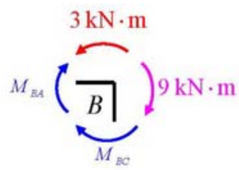
BC 段（$\theta_C = 0$）

$$M_{BC} = \frac{4EI}{L}\theta_B + \frac{2EI}{L}\theta_C \quad \Rightarrow M_{BC} = \frac{2EI}{3}\theta_B$$

$$M_{CB} = \frac{2EI}{L}\theta_B + \frac{4EI}{L}\theta_C \quad \Rightarrow M_{CB} = \frac{EI}{3}\theta_B$$

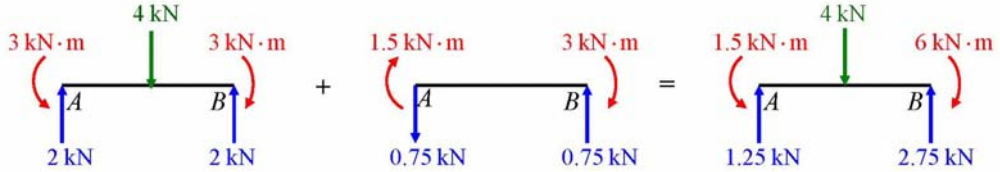


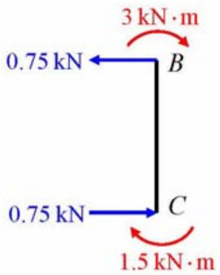
取 B 節點彎矩平衡:　　$M_{BA}+M_{BC}-3+9=0 \quad \Rightarrow \theta_B=-\dfrac{9}{2EI}$



$\therefore \quad M_{AB}=\dfrac{EI}{3}\theta_B=-1.5\,(\mathrm{kN\cdot m})\,,\quad M_{BA}=\dfrac{2EI}{3}\theta_B=-3\,(\mathrm{kN\cdot m})$

$M_{BC}=\dfrac{2EI}{3}\theta_B=-3\,(\mathrm{kN\cdot m})\,,\quad M_{CB}=\dfrac{EI}{3}\theta_B=-1.5\,(\mathrm{kN\cdot m})$





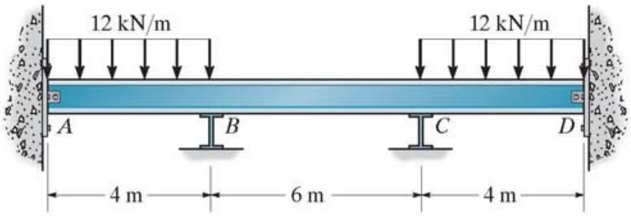
$\therefore \quad M_{AB}=1.5\,(\mathrm{kN\cdot m})$　(逆時鐘方向)

$M_{BA}=6\,(\mathrm{kN\cdot m})$　(順時鐘方向)

$M_{BC}=3\,(\mathrm{kN\cdot m})$　(順時鐘方向)

$M_{CB}=1.5\,(\mathrm{kN\cdot m})$　(順時鐘方向)

3. 如下圖，若 EI 為定值，假設 B 與 C 點為滾支承，A 與 D 點為樞支承，試以彎矩分配法求出 B 及 C 處的彎矩，並繪出該梁的彎矩圖。(20 分)



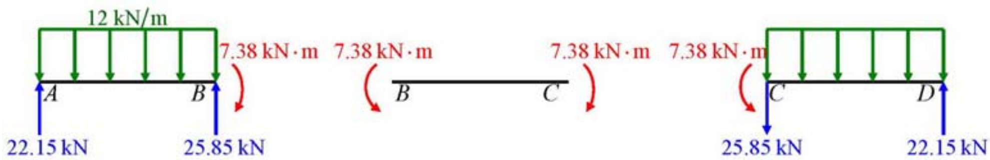
由圖可知此為對稱結構

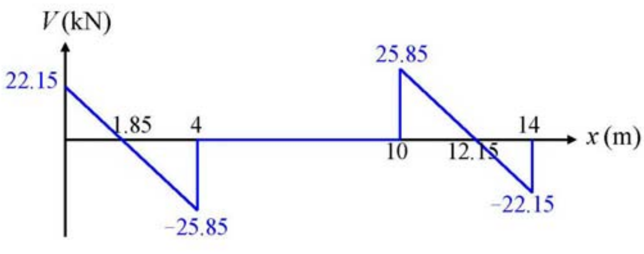
$$(\mathrm{FEM})_{BA} = -\frac{w\ell^2}{8} = -\frac{12\cdot 4^2}{8} = -24\ (\mathrm{kN}\cdot\mathrm{m}),$$

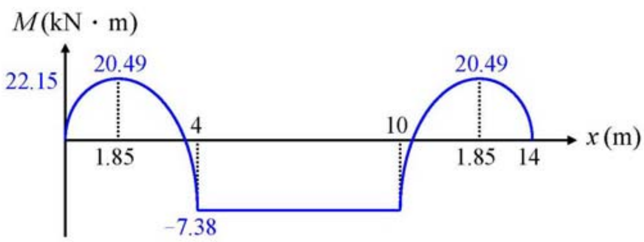
$$k_{BA} = \frac{3}{4}\cdot\frac{4EI}{\ell_{AB}} = \frac{3}{4}EI,\quad k_{BC} = \frac{1}{2}\cdot\frac{4EI}{\ell_{BC}} = \frac{EI}{3}$$

$$\mathrm{DF}_{BA} = \frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{3}} = \frac{9}{13} = 0.6923,\quad \mathrm{DF}_{BC} = \frac{\frac{1}{3}}{\frac{3}{4}+\frac{1}{3}} = \frac{4}{13} = 0.3077$$

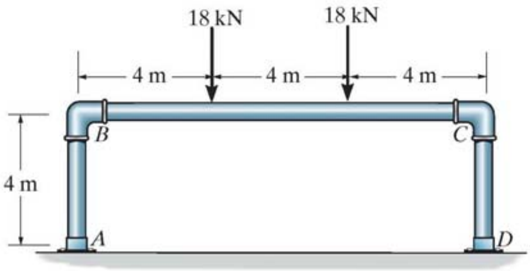
節點	A	B	
構件	AB	BA	BC
DF	1	$\frac{9}{13}$	$\frac{4}{13}$
FEM D1		−24 16.62	 7.38
$\sum M$		−7.38	7.38







4. 如下圖，剛架由固定連接的管子製成，設 EI 為定值，若承受如圖所示之載重，試以彎矩分配法求出在各接點和支承處產生的彎矩。(20 分)



由圖可知此為對稱結構

$$(\text{FEM})_{BC} = \frac{18\cdot 4\cdot 8^2}{12} + \frac{18\cdot 4^2\cdot 8}{12} = 48\ (\text{kN}\cdot\text{m}),$$

$$(\text{FEM})_{CB} = -48\ (\text{kN}\cdot\text{m})$$

$$k_{BA} = \frac{4EI}{\ell_{AB}} = EI,\quad k_{BC} = \frac{1}{2}\cdot\frac{4EI}{\ell_{BC}} = \frac{EI}{6}$$

$$\text{DF}_{BA} = \frac{1}{1+\frac{1}{6}} = \frac{6}{7} = 0.8571,\quad \text{DF}_{BC} = \frac{\frac{1}{6}}{1+\frac{1}{6}} = \frac{1}{7} = 0.1429$$

節點	A	B	
構件	AB	BA	BC
DF	0	$\frac{6}{7}$	$\frac{1}{7}$
FEM D1		 −41.14	48 −6.86
C1	−20.57		
$\sum M$	−20.57	−41.14	41.14

$\therefore\ M_{AB} = 20.57\ (\text{kN}\cdot\text{m})$ (順時鐘方向)

$M_{BA} = 41.14\ (\text{kN}\cdot\text{m})$ (順時鐘方向)

$M_{BC} = 41.14\ (\text{kN}\cdot\text{m})$ (逆時鐘方向)

$M_{CB} = 41.14\ (\text{kN}\cdot\text{m})$ (順時鐘方向)

$M_{CD} = 41.14\ (\text{kN}\cdot\text{m})$ (逆時鐘方向)

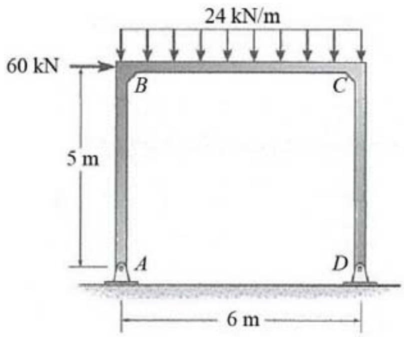
$M_{DC} = 20.57\ (\text{kN}\cdot\text{m})$ (逆時鐘方向)

5. 如下圖，若 EI 為定值

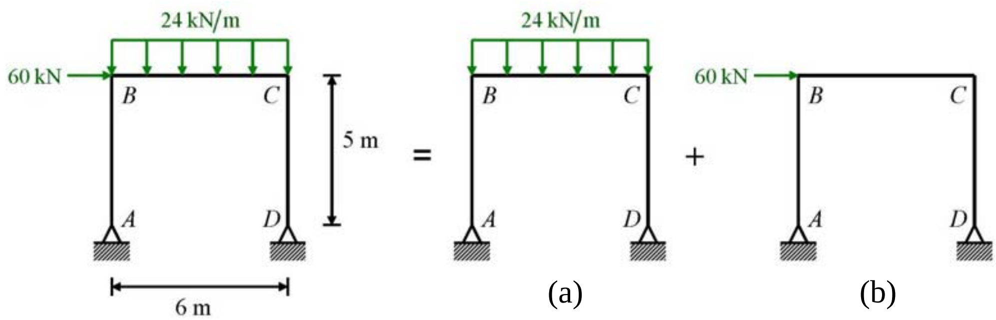
(1) 限以傾角變位法求各端點彎矩與構架的側位移。(16 分)

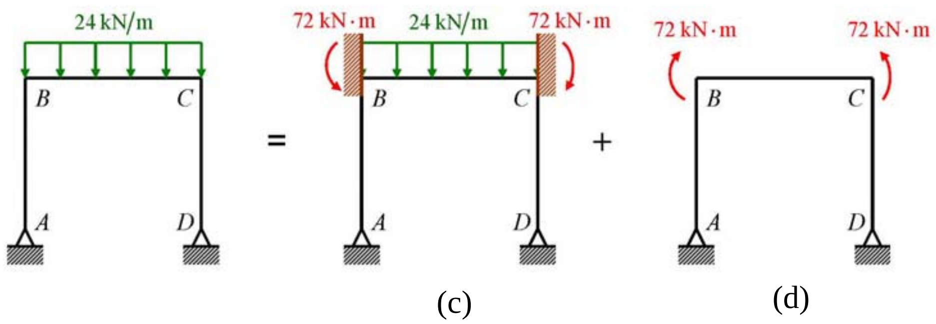
(2) 限以彎矩分配法求各端點彎矩與構架的側位移。(16 分)。

(3) 請畫出此構架的剪力圖與彎矩圖。(8 分)



(1)

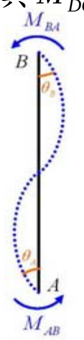




$$(\text{FEM})_{BC} = \frac{w\ell^2}{12} = \frac{24\cdot 6^2}{12} = 72\ (\text{kN}\cdot\text{m}),\quad (\text{FEM})_{CB} = -72\ (\text{kN}\cdot\text{m})$$

先計算圖(d)，由圖可知 A 與 D 端皆為樞接，即 $M_{AB}=0$ 與 $M_{DC}=0$

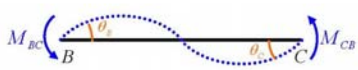
AB 段（$M_{AB}=0$）



$$M_{AB} = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B \quad \Rightarrow \theta_A = -\frac{1}{2}\theta_B$$

$$M_{BA} = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B \quad \Rightarrow M_{BA} = \frac{3EI}{5}\theta_B$$

BC 段



$$M_{BC} = \frac{4EI}{L}\theta_B + \frac{2EI}{L}\theta_C \quad \Rightarrow M_{BC} = \frac{2EI}{3}\theta_B + \frac{EI}{3}\theta_C$$

$$M_{CB} = \frac{2EI}{L}\theta_B + \frac{4EI}{L}\theta_C \quad \Rightarrow M_{CB} = \frac{EI}{3}\theta_B + \frac{2EI}{3}\theta_C$$

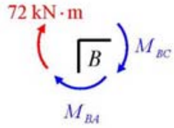
CD 段（$M_{DC}=0$）

$$M_{CD}=\frac{4EI}{L}\theta_C+\frac{2EI}{L}\theta_D \quad \Rightarrow M_{CD}=\frac{4EI}{5}\theta_C+\frac{2EI}{5}\theta_D$$

$$M_{DC}=\frac{2EI}{L}\theta_C+\frac{4EI}{L}\theta_D \quad \Rightarrow \theta_D=-\frac{1}{2}\theta_C$$

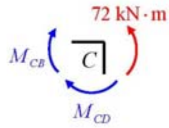
代回 M_{CD} 可得 $M_{CD}=\dfrac{3EI}{5}\theta_C$

取 B 節點彎矩平衡:



$$M_{BA}+M_{BC}+72=0 \quad \Rightarrow \frac{19EI}{15}\theta_B+\frac{EI}{3}\theta_C=-72 \quad \ldots (1)$$

取 C 節點彎矩平衡:



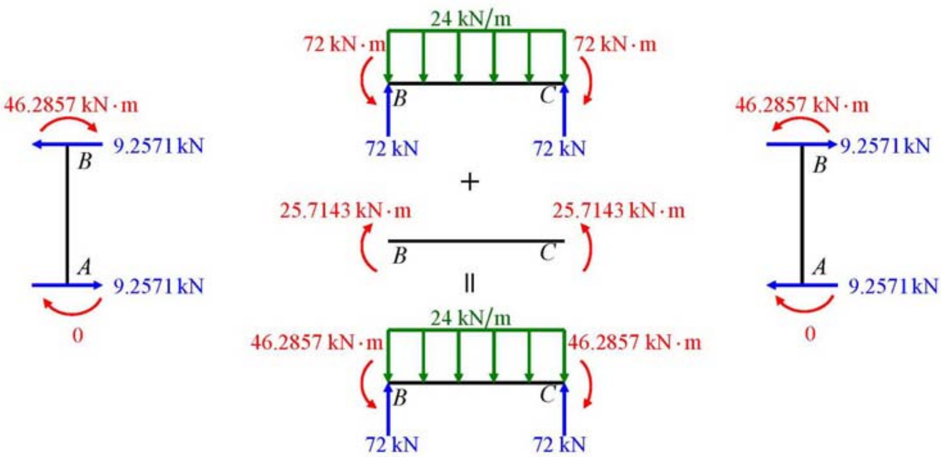
$$M_{CB}+M_{CD}-72=0 \quad \Rightarrow \frac{EI}{3}\theta_B+\frac{19EI}{15}\theta_C=72 \quad \ldots (2)$$

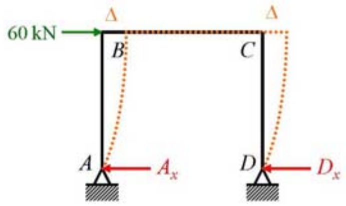
由(1)、(2)可得 $\theta_C=-\theta_B=\dfrac{540}{7EI}$，$\theta_D=-\dfrac{1}{2}\theta_C=-\dfrac{270}{7EI}$

$\therefore\ M_{AB}=0$，$M_{BA}=-\dfrac{324}{7}=-46.2857\ (\text{kN}\cdot\text{m})$，

$M_{BC}=-\dfrac{180}{7}=-25.7143\,(\text{kN}\cdot\text{m})$，$M_{CB}=\dfrac{180}{7}=25.7143\ (\text{kN}\cdot\text{m})$

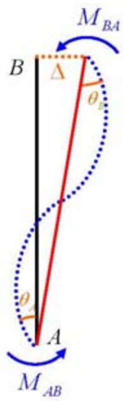
$M_{CD}=\dfrac{324}{7}=46.2857\ (\text{kN}\cdot\text{m})$，$M_{DC}=0$





再計算圖(b)，由圖可知 A 與 D 端皆為樞接，即 $M_{AB}=0$ 與 $M_{DC}=0$，且 AB、CD 段有側向位移 Δ，BC 段無側向位移。

AB 段（$M_{AB}=0$）



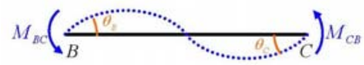
$$M_{AB}=\frac{4EI}{L}\theta_A+\frac{2EI}{L}\theta_B+\frac{6EI}{L}(\frac{\Delta}{L}) \quad \Rightarrow 0=\frac{4EI}{5}\theta_A+\frac{2EI}{5}\theta_B+\frac{6EI}{25}\Delta$$

$$\Rightarrow \theta_A=-(\frac{1}{2}\theta_B+\frac{3}{10}\Delta)$$

$$M_{BA}=\frac{2EI}{L}\theta_A+\frac{4EI}{L}\theta_B+\frac{6EI}{L}(\frac{\Delta}{L}) \quad \Rightarrow M_{BA}=\frac{2EI}{5}\theta_A+\frac{4EI}{5}\theta_B+\frac{6EI}{25}\Delta$$

$$\Rightarrow M_{BA}=\frac{3EI}{5}\theta_B+\frac{3EI}{25}\Delta$$

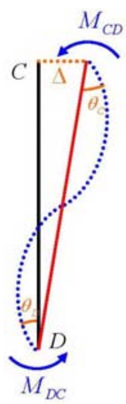
BC 段



$$M_{BC}=\frac{4EI}{L}\theta_B+\frac{2EI}{L}\theta_C \quad \Rightarrow M_{BC}=\frac{2EI}{3}\theta_B+\frac{EI}{3}\theta_C$$

$$M_{CB}=\frac{2EI}{L}\theta_B+\frac{4EI}{L}\theta_C \quad \Rightarrow M_{CB}=\frac{EI}{3}\theta_B+\frac{2EI}{3}\theta_C$$

CD 段（$M_{DC}=0$）



$$M_{CD}=\frac{4EI}{L}\theta_C+\frac{2EI}{L}\theta_D+\frac{6EI}{L}(\frac{\Delta}{L}) \quad \Rightarrow M_{CD}=\frac{4EI}{5}\theta_C+\frac{2EI}{5}\theta_D+\frac{6EI\Delta}{25}$$

$$M_{DC}=\frac{2EI}{L}\theta_C+\frac{4EI}{L}\theta_D+\frac{6EI}{L}(\frac{\Delta}{L}) \quad \Rightarrow \frac{2EI}{5}\theta_C+\frac{4EI}{5}\theta_D+\frac{6EI\Delta}{25}=0$$

$$\Rightarrow \theta_D=-(\frac{1}{2}\theta_C+\frac{3}{10}\Delta)$$

代回 M_{CD} 可得 $M_{CD}=\frac{3EI}{5}\theta_C+\frac{3EI}{25}\Delta$

取 B 節點彎矩平衡:

$$M_{BA}+M_{BC}=0 \quad \Rightarrow \frac{19EI}{15}\theta_B+\frac{EI}{3}\theta_C+\frac{3EI}{25}\Delta=0 \quad \ldots (1)$$

取 C 節點彎矩平衡:

$$M_{CB}+M_{CD}=0 \quad \Rightarrow \frac{EI}{3}\theta_B+\frac{19EI}{15}\theta_C+\frac{3EI}{25}\Delta=0 \quad \ldots (2)$$

由力平衡: $\sum F_x=0 \quad \Rightarrow A_x+D_x=60$

又 $A_x = \dfrac{M_{BA}}{L} = \dfrac{3EI}{25}\theta_B + \dfrac{3EI}{125}\Delta$

$D_x = \dfrac{M_{CD}}{L} = \dfrac{3EI}{25}\theta_C + \dfrac{3EI}{125}\Delta$

$\therefore \quad A_x + D_x = 60 \quad \Rightarrow \dfrac{3EI}{25}\theta_B + \dfrac{3EI}{25}\theta_C + \dfrac{6EI}{125}\Delta = 60 \quad \ldots (3)$

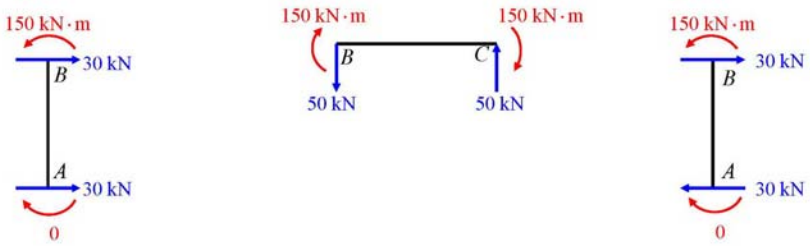
由(1)(2)(3)可得 $\quad \theta_C = \theta_B = -\dfrac{150}{EI}, \quad \Delta = -\dfrac{40}{3}\theta_C = \dfrac{2000}{EI}$

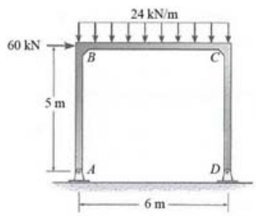
$\therefore \quad M_{BA} = \dfrac{3EI}{5}\theta_B + \dfrac{3EI}{25}\Delta = 150 \text{ (kN}\cdot\text{m)}$

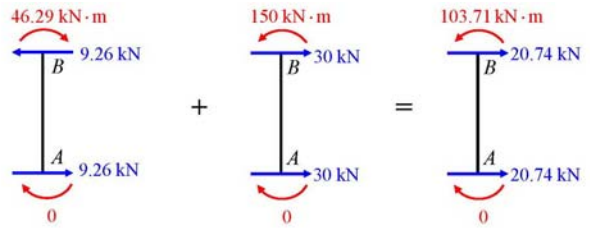
$M_{BC} = \dfrac{2EI}{3}\theta_B + \dfrac{EI}{3}\theta_C = -150 \text{ (kN}\cdot\text{m)}$

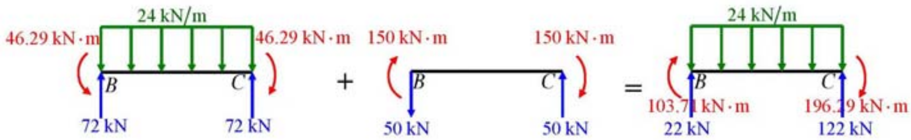
$M_{CB} = \dfrac{EI}{3}\theta_B + \dfrac{2EI}{3}\theta_C = -150 \text{ (kN}\cdot\text{m)}$

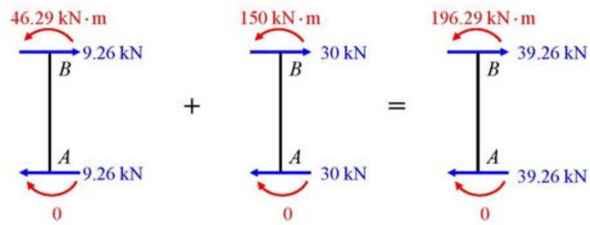
$M_{CD} = \dfrac{3EI}{5}\theta_C + \dfrac{3EI}{25}\Delta = 150 \text{ (kN}\cdot\text{m)}$



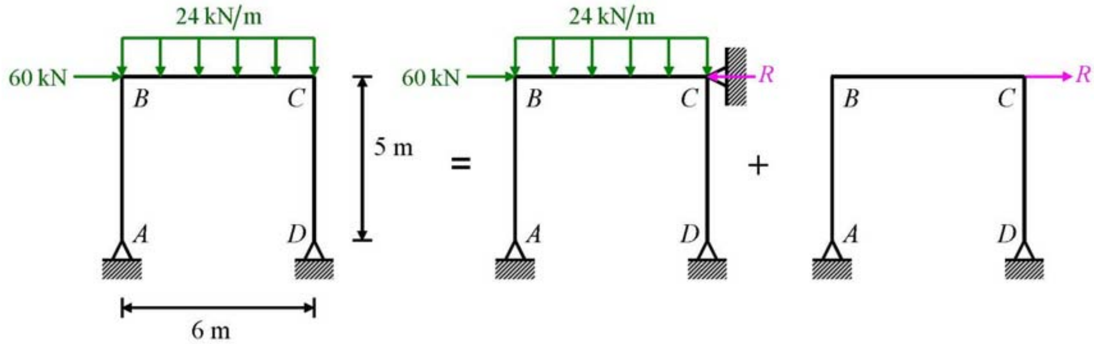








(2)



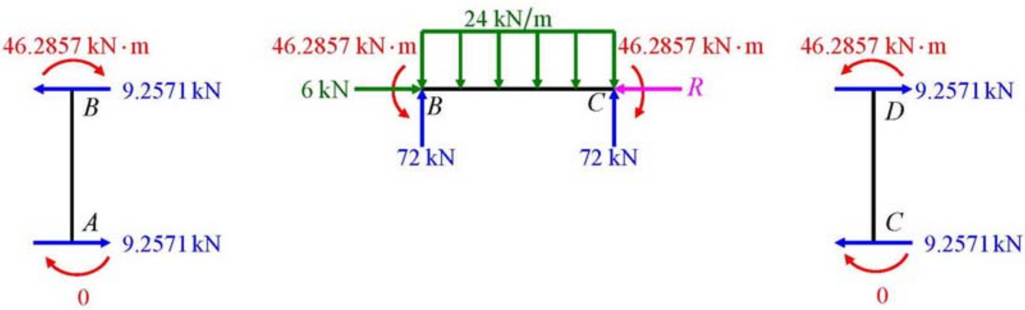
上圖無側向位移唯一可能為對稱，即反力 $R = 60\ (\text{kN})$

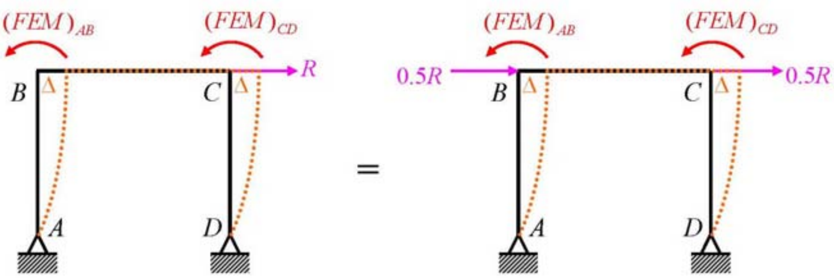
$$(\text{FEM})_{BC} = \frac{w\ell^2}{12} = \frac{24\cdot 6^2}{12} = 72\ (\text{kN}\cdot\text{m}),\quad (\text{FEM})_{CB} = -72\ (\text{kN}\cdot\text{m})$$

$$k_{BA} = \frac{3}{4}\cdot\frac{4EI}{L_{AB}} = \frac{3EI}{5},\quad k_{BC} = \frac{1}{2}\cdot\frac{4EI}{L_{BC}} = \frac{EI}{3}$$

$$DF_{BA} = \frac{k_{BA}}{k_{BA}+k_{BC}} = \frac{9}{14} = 0.6429,\ DF_{BC} = \frac{k_{BC}}{k_{BA}+k_{BC}} = \frac{5}{14} = 0.3571$$

節點	A	B	
構件	AB	BA	BC
DF	1	0.6429	0.3571
FEM			72
D1		−46.2857	−25.7143
$\sum M$		−46.2857	46.2857





由圖可知此為反對稱

$$(\text{FEM})_{BA}=(\text{FEM})_{CD}=\frac{3EI\Delta}{L^2}$$

$$k_{BA}=\frac{3}{4}\cdot\frac{4EI}{L_{AB}}=\frac{3EI}{5}\,,\quad k_{BC}=\frac{3}{2}\cdot\frac{4EI}{L_{BC}}=EI$$

$$DF_{BA}=\frac{k_{BA}}{k_{BA}+k_{BC}}=\frac{3}{8}=0.375\,,\quad DF_{BC}=\frac{k_{BC}}{k_{BA}+k_{BC}}=\frac{5}{8}=0.625$$

節點	A	B	
構件	AB	BA	BC
DF	1	0.375	0.625
FEM		3	
D1		−1.125	−1.875
$\sum M$		1.875	−1.875

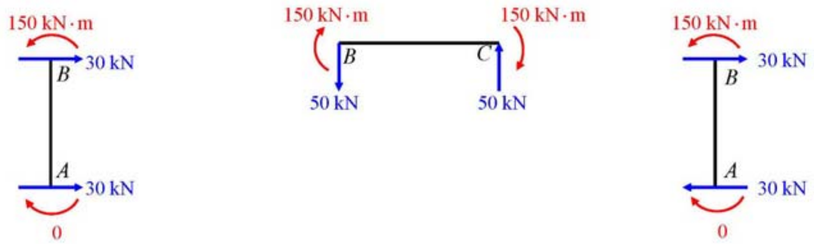
$$\sum F_x=0\quad\Rightarrow 0.375\frac{EI\Delta}{L^2}+0.375\frac{EI\Delta}{L^2}-R=0\qquad\Rightarrow \Delta=\frac{4}{3}\cdot\frac{RL^2}{EI}=\frac{2000}{EI}$$

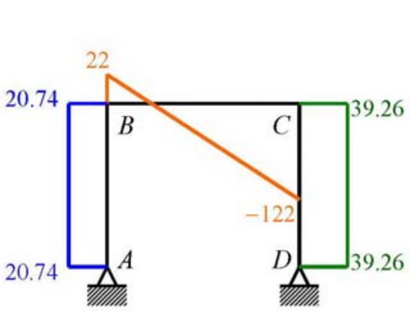
$$M_{BA}=1.875\frac{EI\Delta}{L^2}=150\ (\text{kN}\cdot\text{m})$$

$$M_{BC}=-1.875\frac{EI\Delta}{L^2}=-150\ (\text{kN}\cdot\text{m})$$

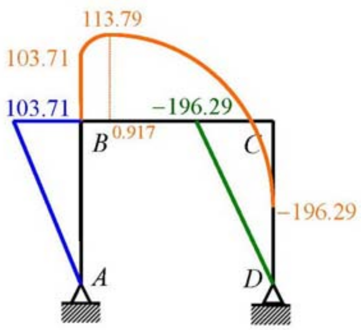
$$M_{CB}=-1.875\frac{EI\Delta}{L^2}=-150\ (\text{kN}\cdot\text{m})$$

$$M_{CD}=1.875\frac{EI\Delta}{L^2}=150\ (\text{kN}\cdot\text{m})$$





剪力圖(單位: kN)



彎矩圖(單位: kN · m)