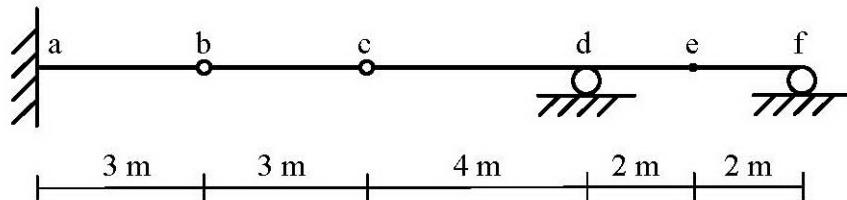


日期：2024 年 12 月 13 日

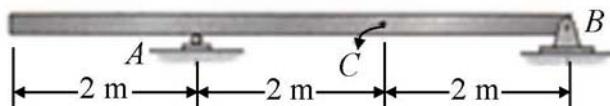
姓名：_____

學號：_____

1. 如下圖所示梁結構，a 點為固定端，d 點及 f 點皆為滾支承，b 點及 c 點解為鉸接。求固定端 a 點垂直反力、固定端 a 點彎矩、d 點支承垂直反力、e 點彎矩及 e 點剪力等五個物理量的影響線。(20 分)
(影響線必須標示數值，只有圖形沒有標示數值者不予計分)

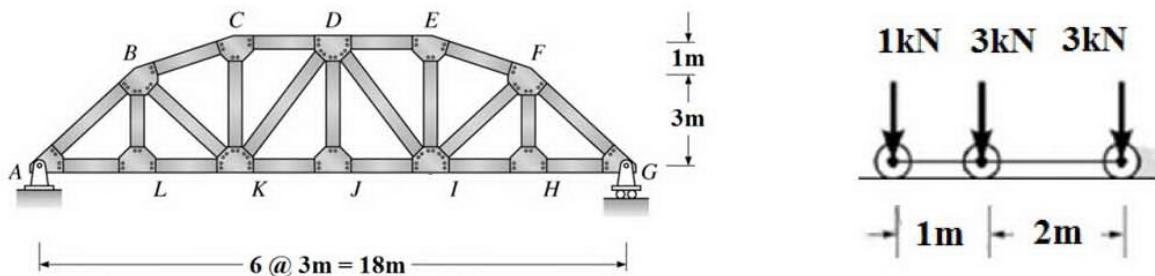


2. 如下圖梁，承受 1.5 kN/m 的均布活載重和 8 kN 的單一集中載重，靜載重為 2 kN/m 。請回答下列問題(A 點是滾接支承，B 點是鉸支承，構件自重不計)
- (1) 繪製 C 點剪力影響線。 (4 分)
 - (2) 繪製 C 點彎矩影響線。 (4 分)
 - (3) 求 C 點最大正剪力。 (6 分)
 - (4) 求 C 點最大正彎矩。 (6 分)

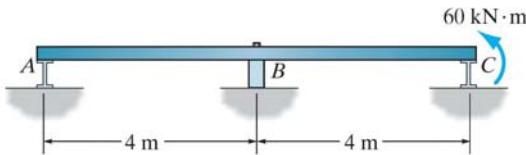


3. 如下圖所示靜定桁架結構，車行橋面位於桁架 AG 線上：

- (1) 試求桿 BK 之桿件力在單一集中移動載重作用下之影響線。 (10 分)
- (2) 當下圖(b)所示移動載重組合，由左向右通過桁架橋面 AG 時，試求桿 BK 在該移動載重組合通過時造成之最大桿力。 (10 分)

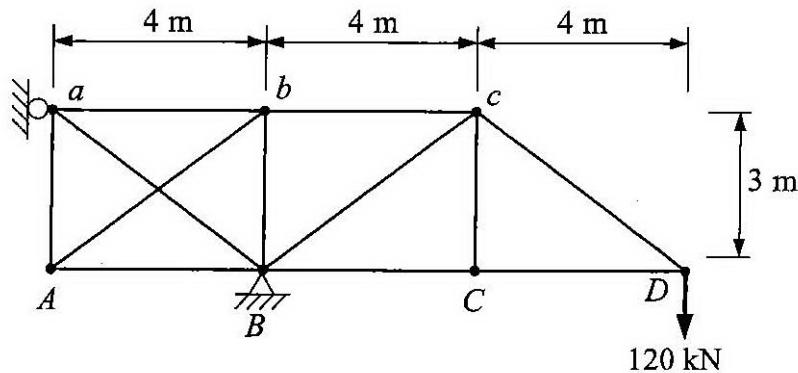


4. 設 A 和 C 為滾支承， B 為樞接支承，且 EI 為常數，求出各支承處的反力，並繪製剪力圖和彎矩圖。(20 分)



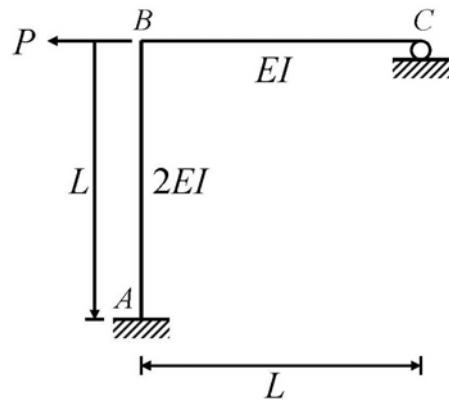
5. 假設下圖所示之桁架所有桿件 $L/A = 1 \text{ (m/cm}^2)$ 、 $E = 200 \times 10^6 \text{ kN/m}^2$ 及熱膨脹係數 $\alpha = 11 \times 10^{-6} \text{ m/m/C}$ ，試分別考慮下列兩種條件，求各桿件之內力，請於答案卷上繪製結構圖，並將各桿件所受內力標於其上，拉力為正，壓力為負。
- (1) 若桁架受圖示之外力作用。(10 分)

- (2) 在不考慮外力的情況下，若桿件 \overline{ab} 、 \overline{bc} 、 \overline{cD} 等溫度上升 40°C ，其餘桿件溫度不變。(10 分)



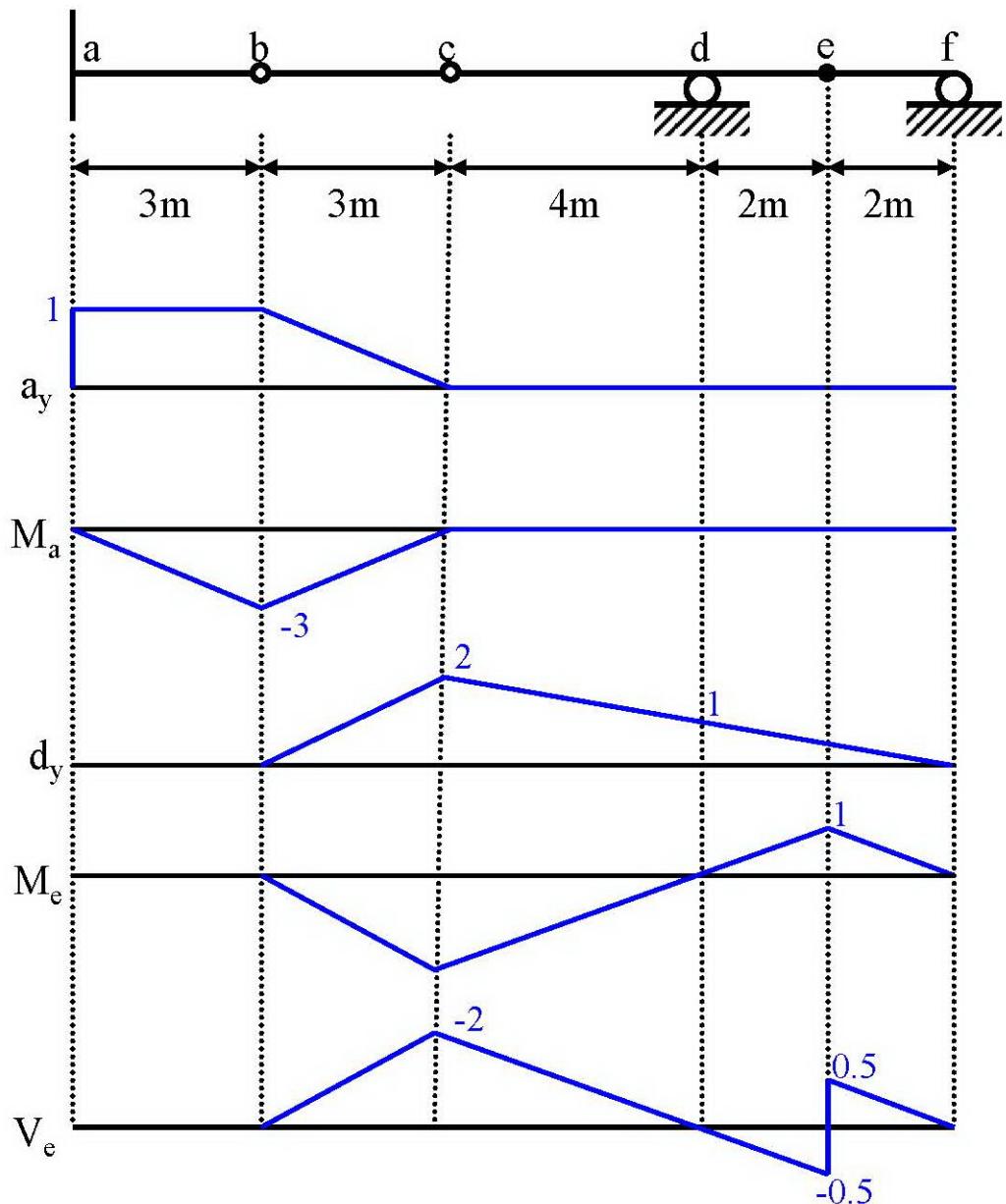
6. 給一構架如下圖所示，試以諧和變位法(力法)求出下列問題

- (1) C 點支承反力，並繪製此構架之彎矩圖。(8 分)
- (2) B 點的位移與旋轉角。(12 分)



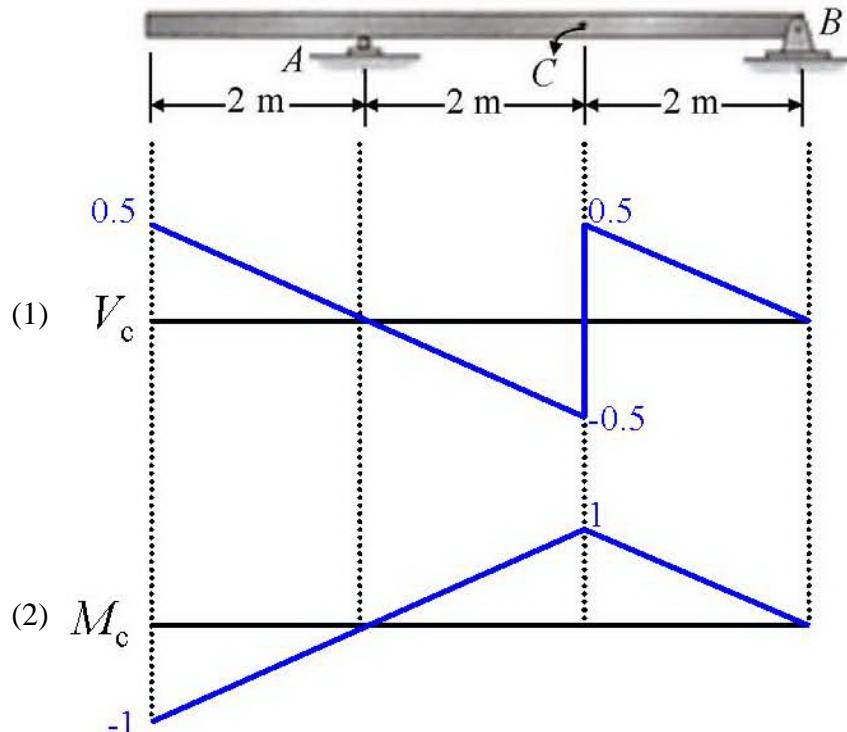
參考解答：

1. 如下圖所示梁結構，a 點為固定端，d 點及 f 點皆為滾支承，b 點及 c 點解為鉸接。求固定端 a 點垂直反力、固定端 a 點彎矩、d 點支承垂直反力、e 點彎矩及 e 點剪力等五個物理量的影響線。(20 分) (106 普考)



2. 如下圖梁，承受 1.5 kN/m 的均布活載重和 8 kN 的單一集中載重，靜載重為 2 kN/m 。請回答下列問題(A 點是滾接支承，B 點是鉸支承，構件自重不計)
- (1) 繪製 C 點剪力影響線。(4 分)
 - (2) 繪製 C 點彎矩影響線。(4 分)
 - (3) 求 C 點最大正剪力。(6 分)
 - (4) 求 C 點最大正彎矩。(6 分)

(111 高考)

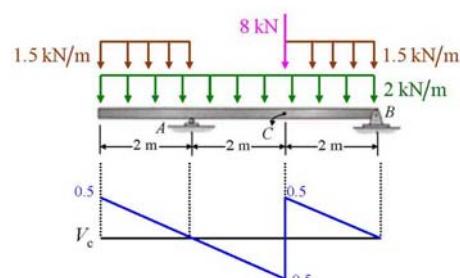


(3) C 點最大剪力

$$(V_C^+)^{max} = 2 \times \left[\left(\frac{1}{2} \times 2 \times \frac{1}{2} \right) \times 2 - \left(\frac{1}{2} \times 2 \times \frac{1}{2} \right) \right]$$

$$+ 1.5 \times \left(\frac{1}{2} \times 2 \times \frac{1}{2} \right) \times 2 + 8 \times \frac{1}{2}$$

$$= 6.5 \text{ (kN)}$$

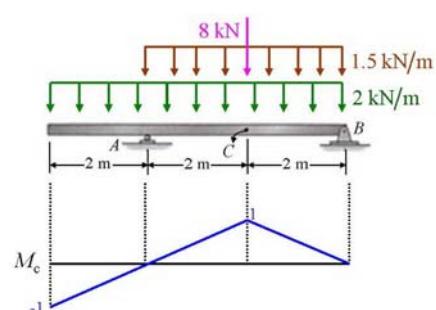


(4) C 點最大彎矩

$$(V_C^+)^{max} = 2 \times \left(\frac{1}{2} \times 4 \times 1 - \frac{1}{2} \times 2 \times 1 \right)$$

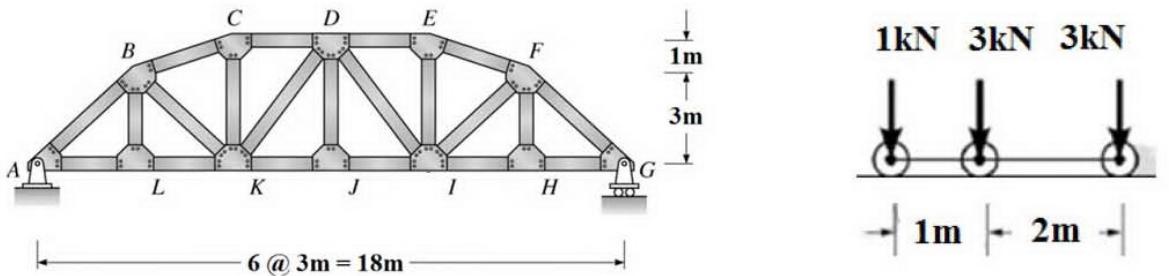
$$+ 1.5 \times \left(\frac{1}{2} \times 4 \times 1 \right) + 8 \times 1$$

$$= 13 \text{ (kN}\cdot\text{m)}$$



3. 如下圖所示靜定桁架結構，車行橋面位於桁架 AG 線上：

- (1) 試求桿 BK 之桿件力在單一集中移動載重作用下之影響線。 (10 分)
- (2) 當下圖(b)所示移動載重組合，由左向右通過桁架橋面 AG 時，試求桿 BK 在該移動載重組合通過時造成之最大桿力。 (10 分) (108 司法特考)



A 節點

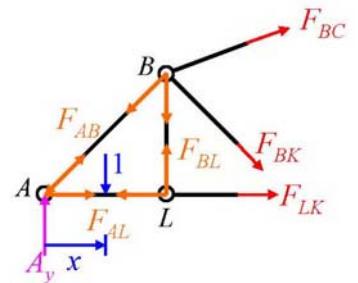
$$\sum F_y = 0 \Rightarrow A_y + F_{AB} \cdot \frac{1}{\sqrt{2}} = 0 \Rightarrow F_{AB} = -\sqrt{2} A_y$$

B 節點

$$\sum F_x = 0 \Rightarrow F_{BC} \cdot \frac{3}{\sqrt{10}} + F_{BK} \cdot \frac{1}{\sqrt{2}} - F_{AB} \cdot \frac{1}{\sqrt{2}} = 0$$

$$\sum F_y = 0 \Rightarrow F_{BC} \cdot \frac{1}{\sqrt{10}} - F_{BK} \cdot \frac{1}{\sqrt{2}} - F_{AB} \cdot \frac{1}{\sqrt{2}} - F_{BL} = 0$$

$$\text{可得 } F_{BK} = -\frac{1}{2} F_{AB} - \frac{3\sqrt{2}}{4} F_{BL} = \frac{\sqrt{2}}{2} A_y - \frac{3\sqrt{2}}{4} F_{BL}$$



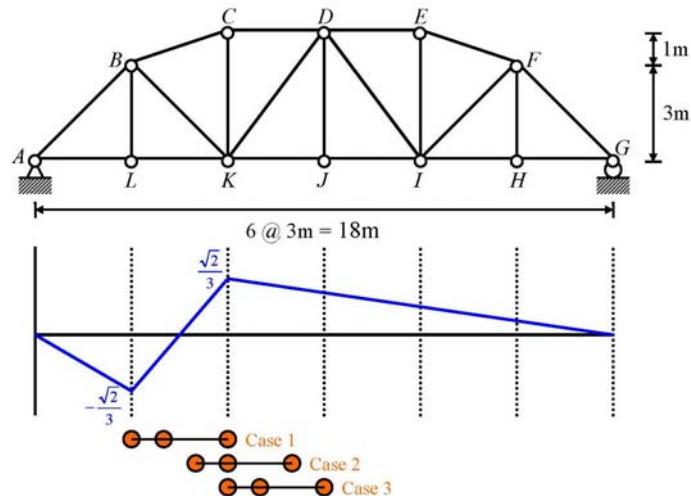
當 $x = 0$, $F_{BK} = 0$

$$\text{當 } x = 3, \quad A_y = \frac{15}{18} = \frac{5}{6}, \quad F_{BL} = 1 \quad \Rightarrow F_{BK} = -\frac{\sqrt{2}}{3}$$

$$\text{當 } x = 6, \quad A_y = \frac{12}{18} = \frac{2}{3}, \quad F_{BL} = 0 \quad \Rightarrow F_{BK} = \frac{\sqrt{2}}{3}$$

$$\text{當 } x = 9, \quad A_y = \frac{9}{18} = \frac{1}{2}, \quad F_{BL} = 0 \quad \Rightarrow F_{BK} = \frac{\sqrt{2}}{4}$$

$$\text{當 } x = 18, \quad A_y = 0, \quad F_{BL} = 0 \quad \Rightarrow F_{BK} = 0$$



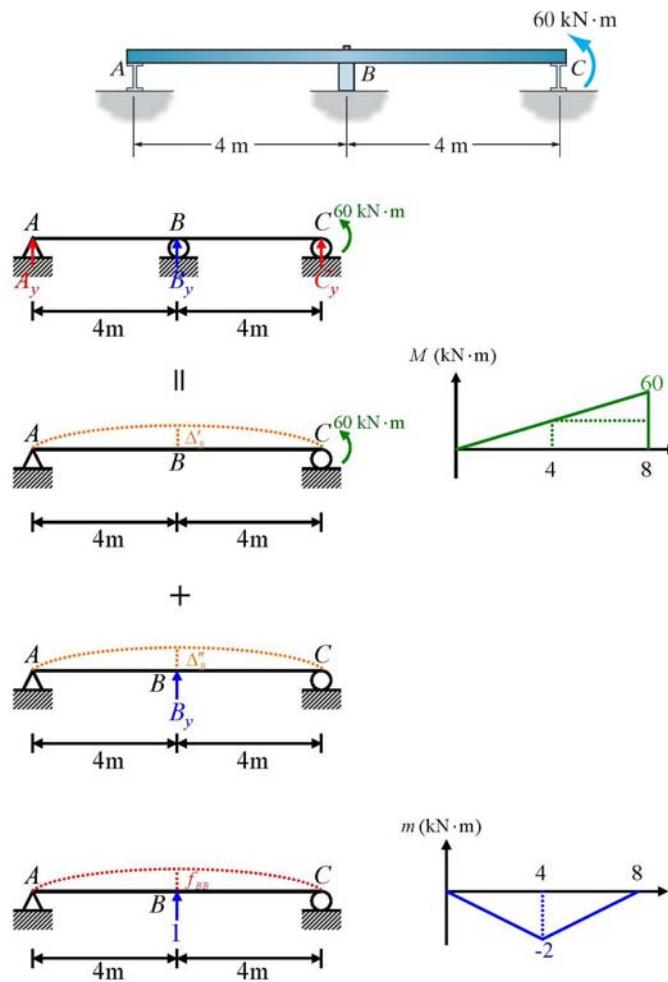
Case 1: $F_{AB} = -\frac{\sqrt{2}}{3} \times 1 + \left(-\frac{\sqrt{2}}{3} + \frac{2\sqrt{2}}{3} \times \frac{1}{3}\right) \times 3 + \frac{\sqrt{2}}{3} \times 3 = \frac{\sqrt{2}}{3}$ (kN)

Case 2: $F_{AB} = \left(\frac{\sqrt{2}}{3} - \frac{2\sqrt{2}}{3} \times \frac{1}{3}\right) \times 1 + \frac{\sqrt{2}}{3} \times 3 + \left(\frac{\sqrt{2}}{3} \times \frac{10}{12}\right) \times 3 = \frac{35\sqrt{2}}{18}$ (kN)

Case 3: $F_{AB} = \frac{\sqrt{2}}{3} \times 1 + \left(\frac{\sqrt{2}}{3} \times \frac{11}{12}\right) \times 3 + \left(\frac{\sqrt{2}}{3} \times \frac{9}{12}\right) \times 3 = 2\sqrt{2}$ (kN)

$$\therefore (F_{AB})_{\max} = 2\sqrt{2}$$
 (kN)

4. 設 A 和 C 為滾支承，B 為樞接支承，且 EI 為常數，求出各支承處的反力，並繪製剪力圖和彎矩圖。(20 分)



$$\text{諧合條件: } \Delta_B = \Delta'_B + \Delta''_B = 0$$

$$\Delta'_B = \frac{1}{EI} \left[\frac{1}{3} \times (-2) \times 30 \times 4 + \frac{1}{2} \times (-2) \times 30 \times 4 + \frac{1}{6} \times (-2) \times 30 \times 4 \right] = -\frac{240}{EI}$$

$$f_{BB} = \frac{1}{EI} \left[\frac{1}{3} \times (-2) \times (-2) \times 4 \right] \times 2 = \frac{32}{3EI}$$

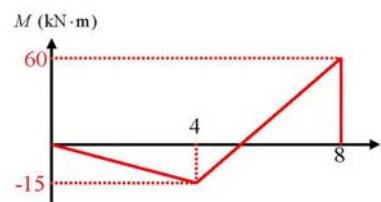
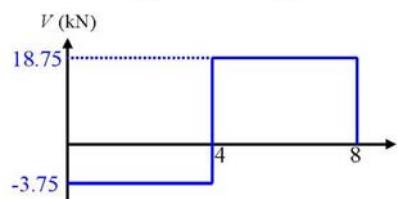
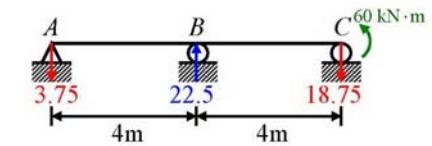
$$\Delta''_B = B_y \times f_{BB}$$

$$\therefore \Delta_B = \Delta'_B + \Delta''_B = 0 \Rightarrow -\frac{240}{EI} + B_y \times \frac{32}{3EI} = 0$$

$$\Rightarrow B_y = \frac{45}{2} \text{ (kN)} = 22.5 \text{ (kN)}$$

$$\sum M_A = 0 \Rightarrow B_y \times 4 + C_y \times 8 + 60 = 0 \Rightarrow C_y = -\frac{75}{4} \text{ (kN)} = -18.75 \text{ (kN)}$$

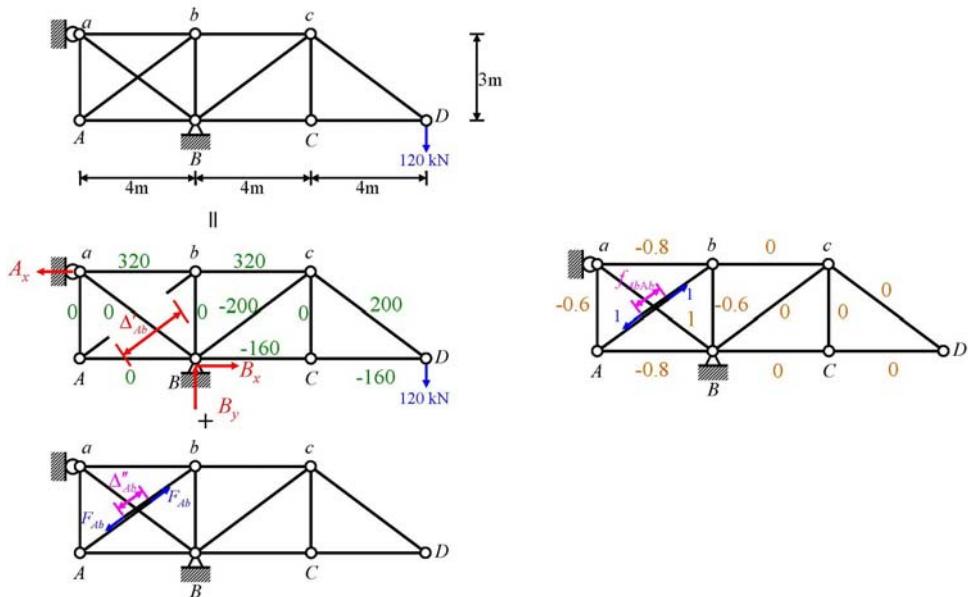
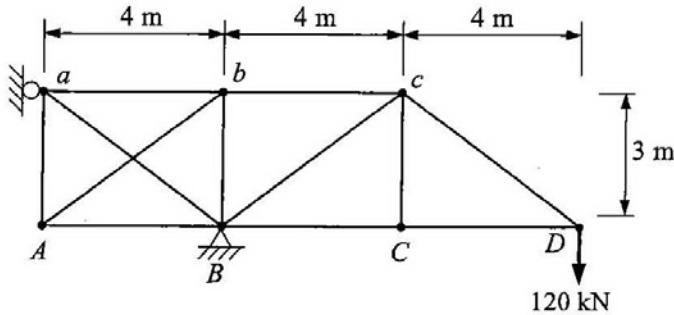
$$\sum F_y = 0 \Rightarrow A_y + B_y + C_y = 0 \Rightarrow A_y = -\frac{15}{4} \text{ (kN)} = -3.75 \text{ (kN)}$$



5. 假設下圖所示之桁架所有桿件 $L/A = 1 \text{ (m/cm}^2)$ 、 $E = 200 \times 10^6 \text{ kN/m}^2$ 及熱膨脹係數 $\alpha = 11 \times 10^{-6} \text{ m/m/}^\circ\text{C}$ ，試分別考慮下列兩種條件，求各桿件之內力，請於答案卷上繪製結構圖，並將各桿件所受內力標於其上，拉力為正，壓力為負。
- (1) 若桁架受圖示之外力作用。(10 分)

- (2) 在不考慮外力的情況下，若桿件 ab 、 bc 、 cD 等溫度上升 40°C ，其餘桿件溫度不變。(10 分)

(106 中央土木)



$$L/A = 1 \text{ (m/cm}^2) = 10^4 \text{ (m/m}^2)$$

$$\text{合規條件: } \Delta_{Ab} = \Delta'_{Ab} + \Delta''_{Ab} = 0$$

$$\sum M_B = 0 \Rightarrow A_x = 320 \text{ (kN)} \quad \sum F_x = 0 \Rightarrow B_x = 320 \text{ (kN)}$$

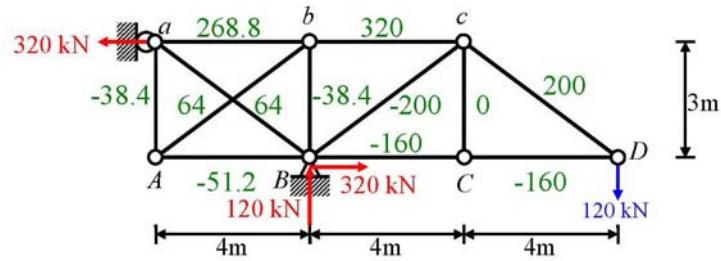
$$\sum F_y = 0 \Rightarrow B_y = 120 \text{ (kN)}$$

$$(1) \Delta'_{Ab} = \sum \frac{nNL}{AE} = \frac{-0.8 \times 320}{200 \times 10^2} = -1.28 \times 10^{-2} \text{ (m)}$$

$$f_{AbAb} = \sum \frac{n^2 L}{AE} = \frac{(-0.8)^2 + (-0.6)^2 + 1^2}{200 \times 10^2} \times 2 = 2 \times 10^{-4} \text{ (m/kN)}$$

$$\Delta''_{Ab} = F_{Ab} \times f_{AbAb}$$

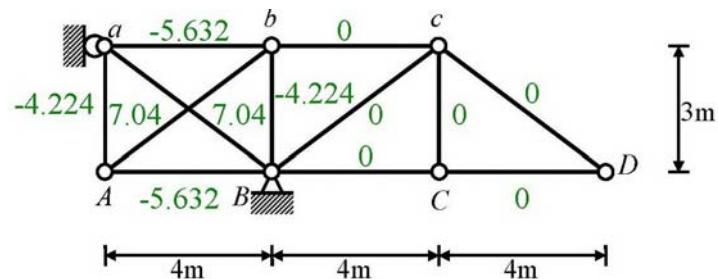
$$\Delta'_{Ab} + \Delta''_{Ab} = 0 \Rightarrow -1.28 \times 10^{-2} + F_{Ab} \times 2 \times 10^{-4} = 0 \Rightarrow F_{Ab} = 64 \text{ (kN)}$$



$$(2) \quad \delta_T = \alpha \times \Delta T \times L$$

$$\Delta'_{Ab} = \sum n \times \delta_T = (-0.8) \times (11 \times 10^{-6} \times 40 \times 4) = -1.408 \times 10^{-3} \text{ (m)}$$

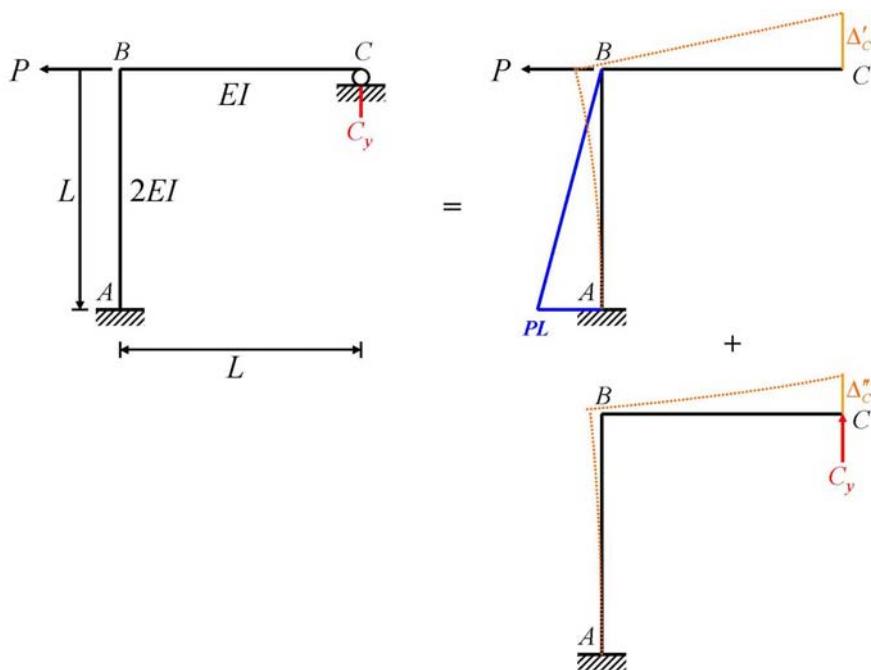
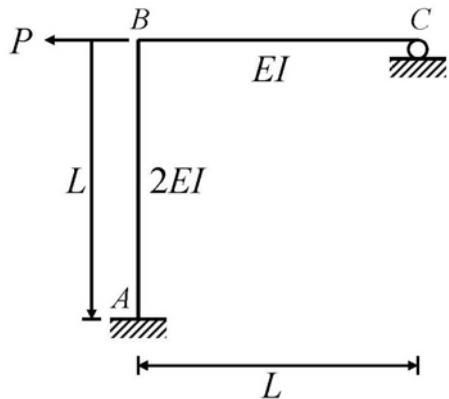
$$\Delta'_{Ab} + \Delta''_{Ab} = 0 \Rightarrow -1.408 \times 10^{-3} + F_{Ab} \times 2 \times 10^{-4} = 0 \Rightarrow F_{Ab} = 7.04 \text{ (kN)}$$



6. 給一構架如下圖所示，試以諧和變位法(力法)求出下列問題

(1) C 點支承反力，並繪製此構架之彎矩圖。(8 分)

(2) B 點的位移與旋轉角。(12 分)



$$\text{諧合條件: } \Delta_c = \Delta'_c + \Delta''_c = 0$$

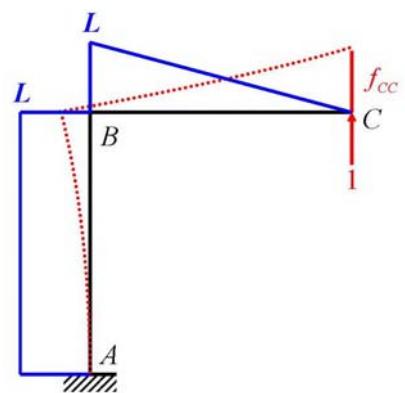
$$\Delta'_c = \frac{1}{2EI} \times \left(\frac{1}{2} \times L \times PL \times L\right) = \frac{PL^3}{4EI}$$

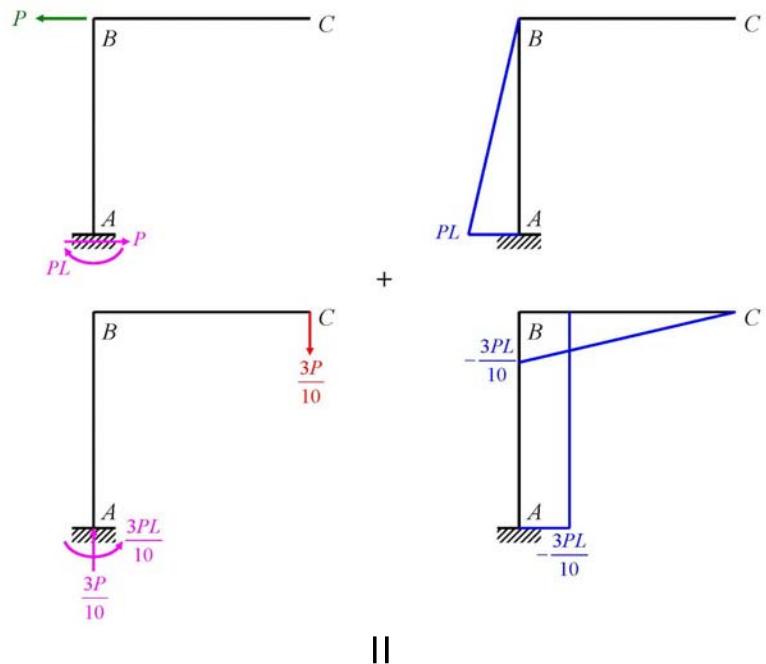
$$f_{cc} = \frac{1}{EI} \times \left(\frac{1}{3} \times L \times L \times L\right) + \frac{1}{2EI} \times (L \times L \times L) = \frac{5L^3}{6EI}$$

$$\Delta''_c = C_y \times f_{cc}$$

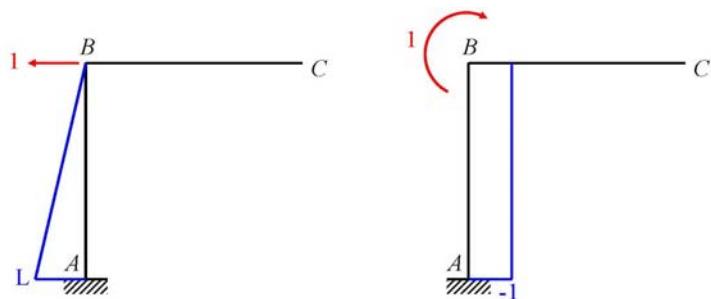
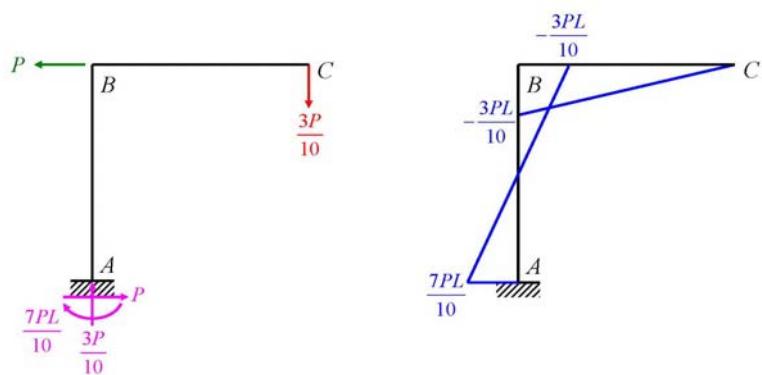
$$\therefore \Delta_c = \Delta'_c + \Delta''_c = 0 \Rightarrow \frac{PL^3}{4EI} + C_y \times \frac{5L^3}{6EI} = 0$$

$$\Rightarrow C_y = -\frac{3P}{10}$$





||



$$\Delta_B = \frac{1}{2EI} \left[\frac{1}{3} \times L \times PL \times L + \frac{1}{2} \times L \times \left(-\frac{3PL}{10} \right) \times L \right] = \frac{11PL^3}{120EI}$$

Δ_B 為正，表示移動方向與單位力方向相同，故為向左移動

$$\theta_B = \frac{1}{2EI} \left[\frac{1}{2} \times (-1) \times PL \times L + (-1) \times \left(-\frac{3PL}{10} \right) \times L \right] = -\frac{PL^2}{10EI}$$

θ_B 為負，表示轉動方向與單位彎矩方向相反，故為逆時鐘方向轉動