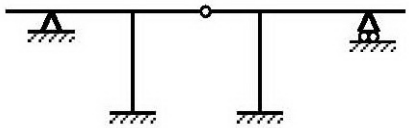
國立臺灣海洋大學河海工程學系 2024 結構學 3A 班第一次大考參考解答

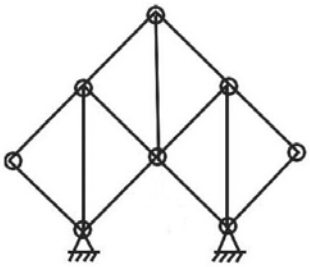
日期：2024 年 11 月 08 日　　姓名：＿＿＿＿＿＿　　學號：＿＿＿＿＿＿

1. 試判斷下列結構之穩定性(stability)與靜定性(static determinacy)。若為不穩定，則說明不穩定的原因;若為靜不定，說明其靜不定次數。(20%)

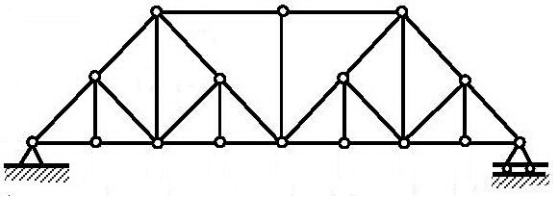
(a) 梁



(b) 構架



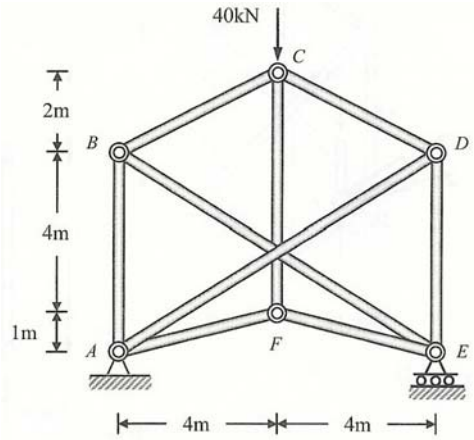
(c) 桁架



(d) 桁架

(98 普考、96 地特三等、106 地特四等)

2. 試求解下圖複雜桁架的 *CF* 桿軸力。 (20%)

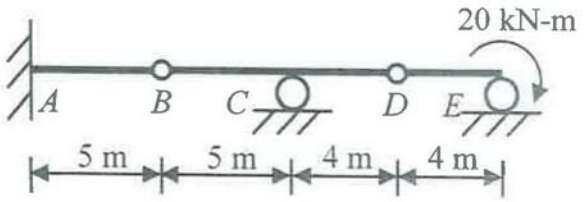


3. 一連續梁幾何與受力情形如下圖所示，各桿件 *EI* 為定值。

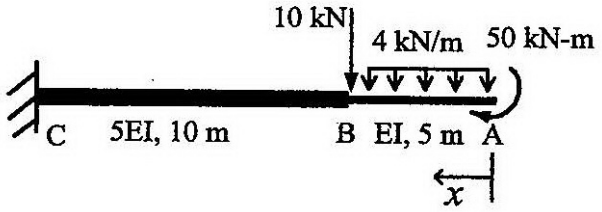
(1) 繪出此連續梁的剪力圖與彎矩圖。 (10%)

(2) 以共軛梁法求 *B* 點與 *D* 點的垂直變位，以及 *E* 點的轉角。 (10%)

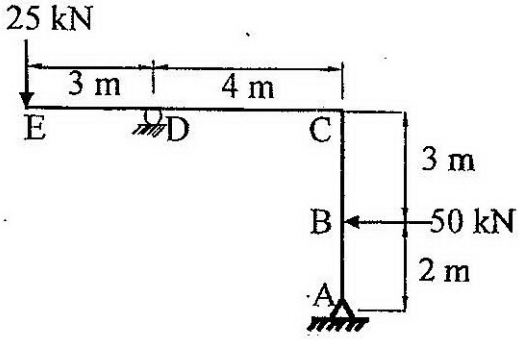
(使用其他方法不予給分)　　(111 台科大營建)



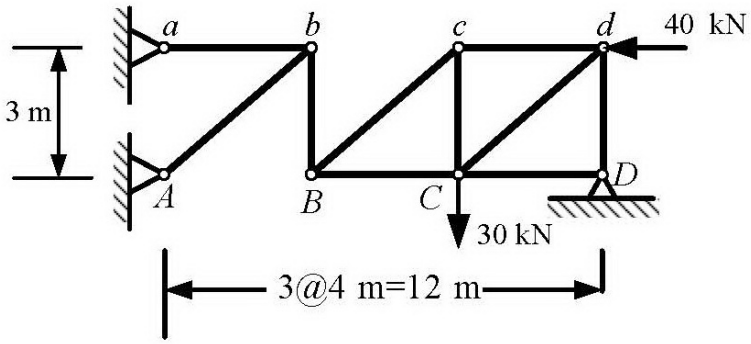
4. 如下圖所示，全長 15m 具有兩種斷面($I_{AB}=I$，$I_{BC}=5I$)之懸臂梁，已知 $E=200\ \text{GPa}$，$I=3\times10^{9}\ \text{mm}^4$，試以單位力法求取 A 點之垂直變位。 (20%)
（以其他方法作答者一律不予以計分） (112 台大土木)



5. 如下圖所示之剛架，A 點為鉸接支承，D 點為滾支承，利用虛功法 (Virtual Work Method) 求 C 點水平位移及繪此剛架彎矩圖 (各桿件之彈性模數 E、慣性矩 I 均相同） (20%) （109 台大土木）
注意:未依指定方法作答，整題以零分計。



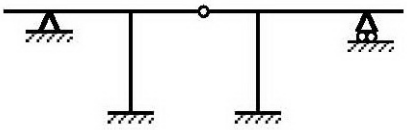
6. 給一桁架如下圖所示，假設所有桿件的 $EA=10^5$ kN，試以卡氏第二定理求解 B 點的水平與垂直位移。 (20%) （107 地特四等)



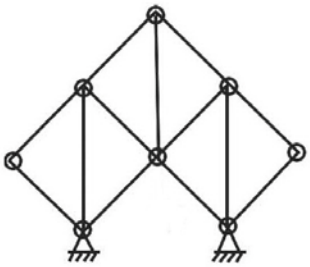
參考解答:

1. 試判斷下列結構之穩定性(stability)與靜定性(static determinacy)。若為不穩定，則說明不穩定的原因;若為靜不定，說明其靜不定次數。(20%)

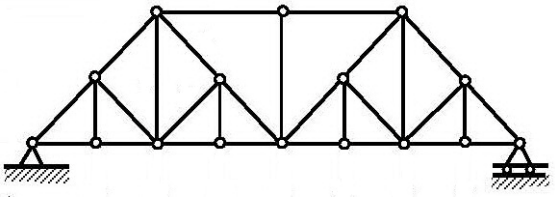
(a) 梁



(b) 構架



(c) 桁架



(d) 桁架

(98 普考、96 地特三等、106 地特四等)

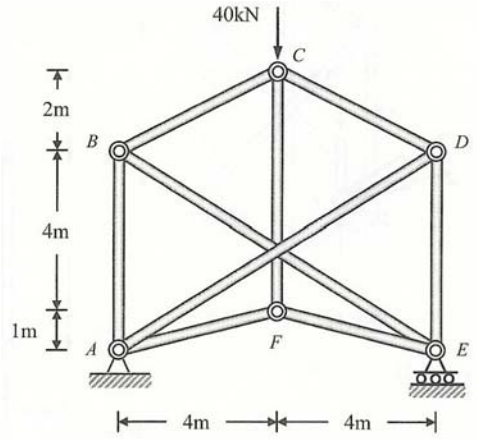
(a) 不穩定，反力數不足 (2 個圈，支承-5，內銷-2，$2\times3-5-2=-1$)

(b) 5 度靜不定 (3 個圈，支承-3，內銷-1，$3\times3-3-1=5$)

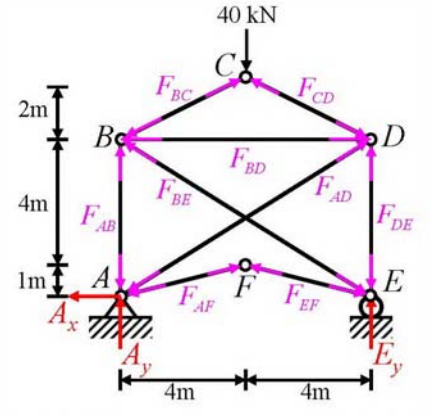
(c) 1 度靜不定 ($b=13,\ r=4,\ j=8,\ b+r-2j=1$)

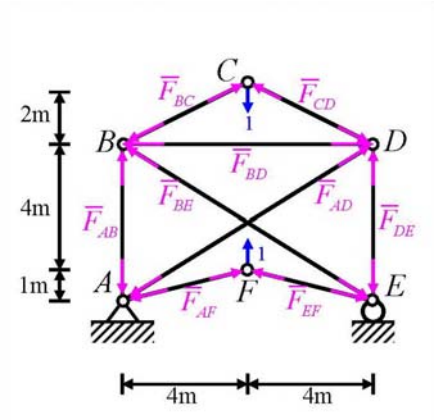
(d) 靜定 ($b=29,\ r=3,\ j=16,\ b+r-2j=0$)

2. 試求解下圖複雜桁架的 CF 桿軸力。 (20%)



用替代桿件法可將問題分解成兩部份





由左圖

由對稱性可得 $A_y = E_y = 25\,(\text{kN})$

$\sum F_x = 0 \;\Rightarrow A_x = 0$

由節點 F 取 $\sum F_y = 0$ 與 $\sum F_x = 0$ 可知 $F_{AF} = F_{EF} = 0$

由節點 E 取 $\sum F_x = 0$ 可知 $F_{BE} = 0$

$\sum F_y = 0$ 可知 $F_{DE} = E_y = 20\,(\text{kN})$

由節點 A 取 $\sum F_x = 0$ 可知 $F_{AD} = 0$

$\sum F_y = 0$ 可知 $F_{AB} = A_y = 20\,(\text{kN})$

由節點 C 取 $\sum F_y = 0$ 與 $\sum F_x = 0$ 可知 $F_{BC} = F_{CD} = 20\sqrt{5}\,(\text{kN})$

由節點 B 取 $\sum F_x = 0$ 可知 $F_{BD} = -40\,(\text{kN})$

由右圖

由節點 C 取 $\sum F_y = 0$ 與 $\sum F_x = 0$ 可知 $\overline{F}_{BC} = \overline{F}_{CD} = \dfrac{\sqrt{5}}{2}\,(\text{kN})$

由節點 F 取 $\sum F_y = 0$ 與 $\sum F_x = 0$ 可知 $\overline{F}_{AF} = \overline{F}_{EF} = -\dfrac{\sqrt{17}}{2}\,(\text{kN})$

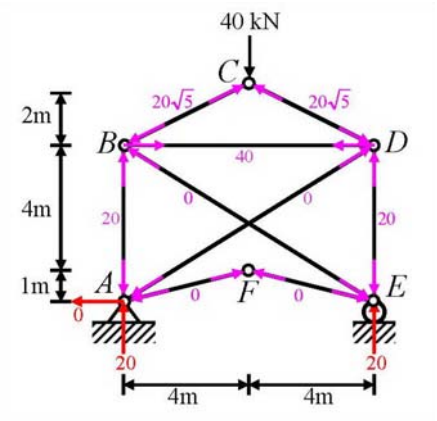
由節點 E 取 $\sum F_x = 0$ 可知 $\overline{F}_{EF} \cdot \dfrac{4}{\sqrt{17}} + \overline{F}_{BE} \cdot \dfrac{8}{\sqrt{89}} = 0 \Rightarrow \overline{F}_{BE} = \dfrac{\sqrt{89}}{4}\,(\text{kN})$

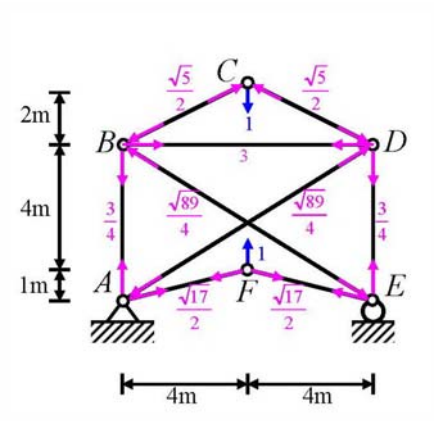
$\sum F_y = 0$ 可知 $F_{DE} = -\dfrac{3}{4}\,(\text{kN})$

由對稱性可得 $\overline{F}_{AB} = F_{DE} = -\dfrac{3}{4}\,(\text{kN})$

$\overline{F}_{AD} = \overline{F}_{BE} = \dfrac{\sqrt{89}}{4}\,(\text{kN})$

由節點 B 取 $\sum F_x = 0$ 可知 $\overline{F}_{BD} = -3\,(\text{kN})$





∵ 原無 BD 桿

$\therefore\ 40 + 3 \cdot F_{CF} = 0 \quad \Rightarrow F_{CF} = -\dfrac{40}{3}$

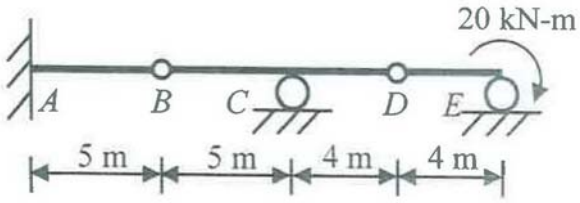
CF 桿為大小為 $\dfrac{40}{3}$ kN (壓)

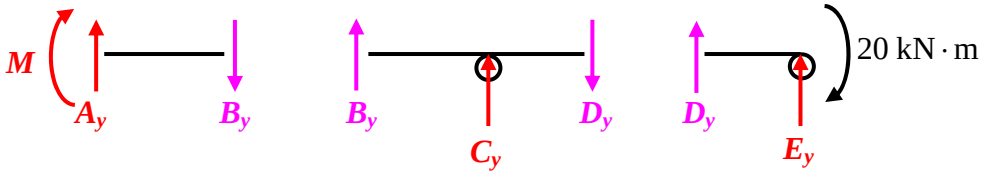
3. 一連續梁幾何與受力情形如下圖所示，各桿件 EI 為定值。

(1) 繪出此連續梁的剪力圖與彎矩圖。(10%)

(2) 以共軛梁法求 B 點與 D 點的垂直變位，以及 E 點的轉角。 (10%)

(使用其他方法不予給分) (111 台科大營建)





DE 段: $\sum M_E = 0 \Rightarrow D_y = -5\,(\text{kN})$

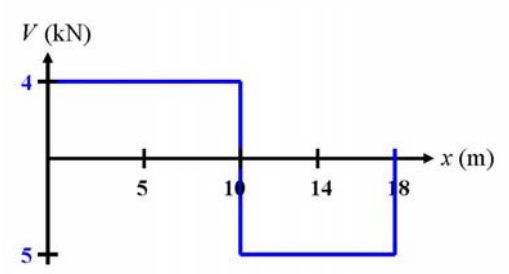
$\sum F_y = 0 \Rightarrow E_y = 5\,(\text{kN})$

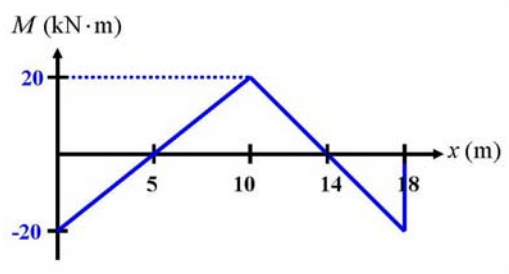
BD 段: $\sum M_C = 0 \Rightarrow B_y = 4\,(\text{kN})$

$\sum F_y = 0 \Rightarrow C_y = -9\,(\text{kN})$

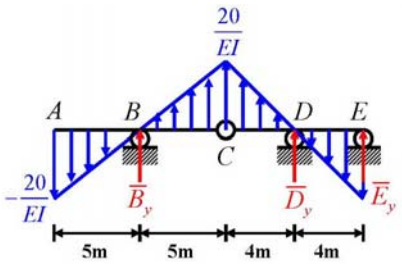
AB 段: $\sum F_y = 0 \Rightarrow A_y = 4\,(\text{kN})$

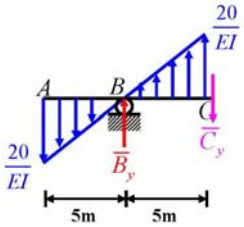
$\sum M_A = 0 \Rightarrow M = -20\,(\text{kN})$





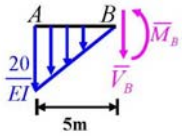
共軛梁:





$$\sum M_B = 0 \quad \Rightarrow \frac{1}{2}\cdot 5 \cdot \frac{20}{EI}\cdot\frac{10}{3}\cdot 2 - \overline{C}_y \cdot 5 = 0$$

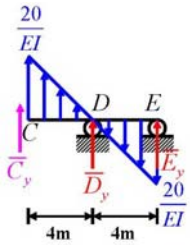
$$\Rightarrow \overline{C}_y = \frac{200}{3EI}$$



$$\sum M_B = 0 \quad \Rightarrow \frac{1}{2}\cdot 5 \cdot \frac{20}{EI}\cdot\frac{10}{3} + \overline{M}_B = 0$$

$$\Rightarrow \overline{M}_B = -\frac{500}{3EI}$$

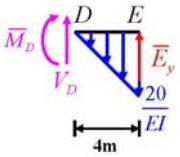
$$\Delta_B = \overline{M}_B = \frac{500}{3EI} \ (\downarrow)$$



$$\sum M_D = 0 \quad \Rightarrow \frac{1}{2}\cdot 4 \cdot \frac{20}{EI}\cdot\frac{8}{3}\cdot 2 + \overline{C}_y \cdot 4 - \overline{E}_y \cdot 4 = 0$$

$$\Rightarrow \overline{E}_y = \frac{120}{EI}$$

$$\theta_E = \overline{E}_y = \frac{120}{EI} \ (\text{順轉})$$

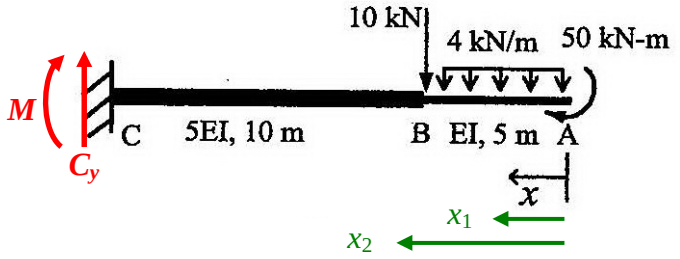


$$\sum M_D = 0 \quad \Rightarrow \frac{1}{2}\cdot 4 \cdot \frac{20}{EI}\cdot\frac{8}{3} - \overline{E}_y \cdot 4 + \overline{M}_D = 0$$

$$\Rightarrow \overline{M}_D = \frac{1120}{3EI}$$

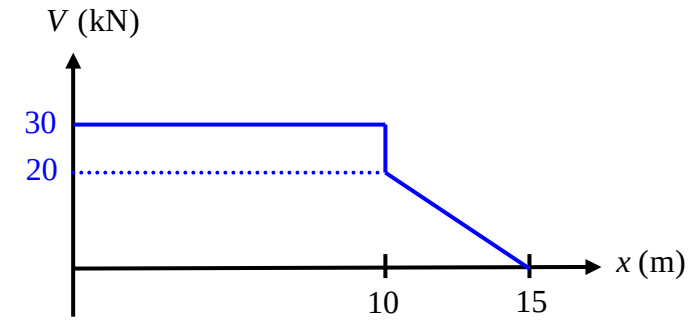
$$\Delta_D = \overline{M}_D = \frac{1120}{3EI} \ (\uparrow)$$

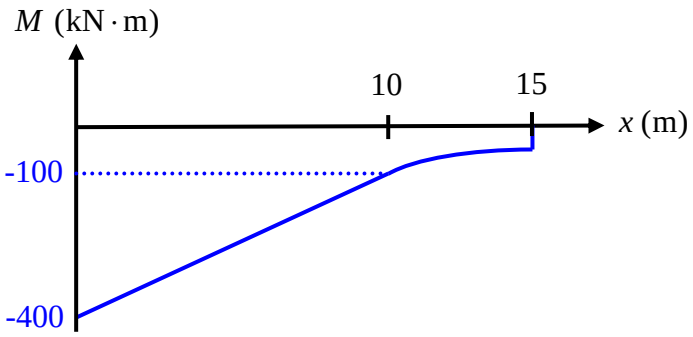
4. 如下圖所示，全長 15m 具有兩種斷面($I_{AB}=I$，$I_{BC}=5I$)之懸臂梁，已知 $E=200\ \text{GPa}$，$I=3\times10^{9}\ \text{mm}^4$，試以單位力法求取 A 點之垂直變位。 (20%)
（以其他方法作答者一律不予以計分） (112 台大土木)

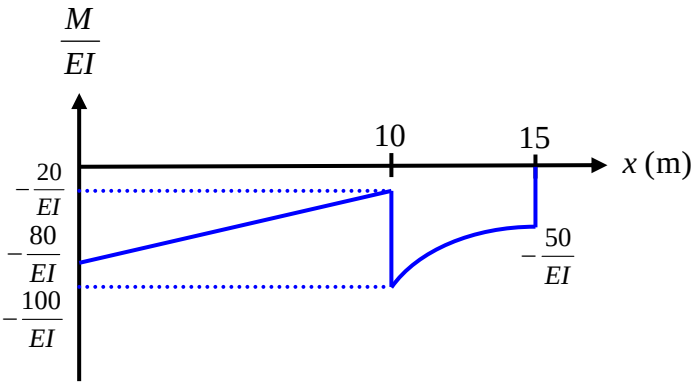


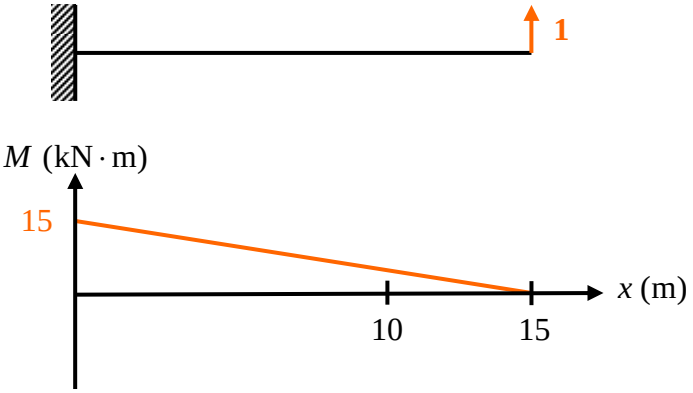
$$\sum M_C = 0 \quad \Rightarrow 10\cdot10+4\cdot5\cdot12.5+50+M=0 \quad \Rightarrow M=-400\ (\text{kN}\cdot\text{m})$$

$$\sum F_y = 0 \quad \Rightarrow C_y = 10+4\cdot5=30\ (\text{kN})$$









$$1\cdot\Delta_A=\int_0^5 x_1\cdot\frac{-50-2x_1^2}{EI}dx_1+\int_5^{15} x_2\cdot\frac{-20-6(x_2-5)}{EI}dx_2$$

$$\Rightarrow\Delta_A=\frac{1}{EI}\int_0^5 x_1\cdot(-50-2x_1^2)\,dx_1+\frac{1}{EI}\int_5^{15} x_2\cdot[-20-6(x_2-5)]\,dx_2$$

$$\Rightarrow\Delta_A=-\frac{1875}{2EI}-\frac{5500}{EI}=-\frac{12875}{EI}=-\frac{6437.5\,(\text{kN}\cdot\text{m}^3)}{EI}$$

$$=-\frac{6437.5}{200\cdot10^6\cdot3\cdot10^{-3}}$$

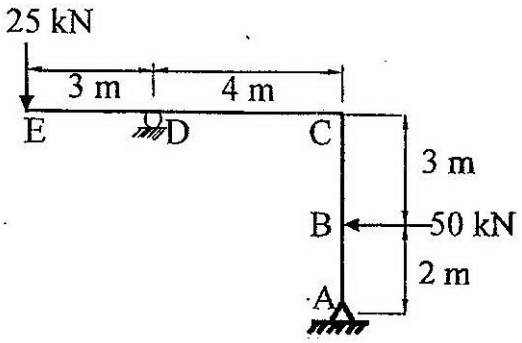
$$=-0.01073\,(\text{m})=-10.73\,(\text{mm})$$

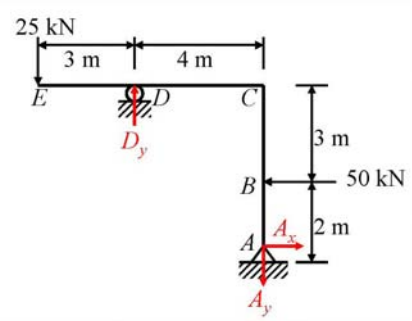
負號表示位移方向與虛單位力作用方向相反

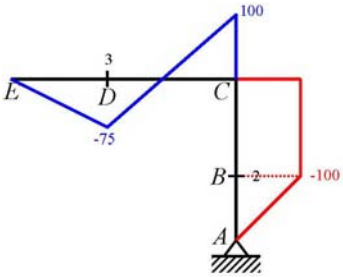
即位移方向為向下

5. 如下圖所示之剛架，A 點為鉸接支承，D 點為滾支承，利用虛功法 (Virtual Work Method) 求 C 點水平位移及繪此剛架彎矩圖 (各桿件之彈性模數 E、慣性矩 I 均相同) (20%) （109 台大土木）

注意：未依指定方法作答，整題以零分計。







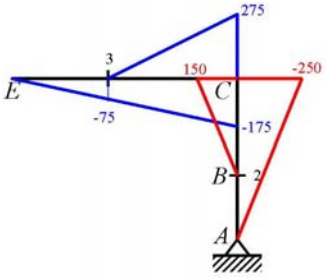
彎矩圖

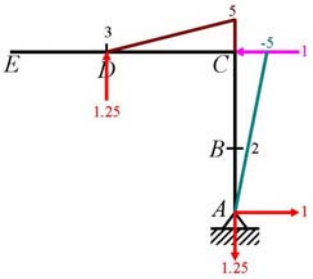
$$\sum F_x = 0 \quad \Rightarrow A_x = 50\,(\text{kN})$$

$$\sum M_A = 0 \quad \Rightarrow 50\cdot 2 + 25\cdot 7 - D_y \cdot 4 = 0 \quad \Rightarrow D_y = \frac{275}{4} = 68.75\,(\text{kN})$$

$$\sum F_y = 0 \quad \Rightarrow A_y = \frac{175}{4} = 43.75\,(\text{kN})$$

可利用疊加法可將彎矩圖拆解如下左圖，給虛單位力所得彎矩圖如下右圖



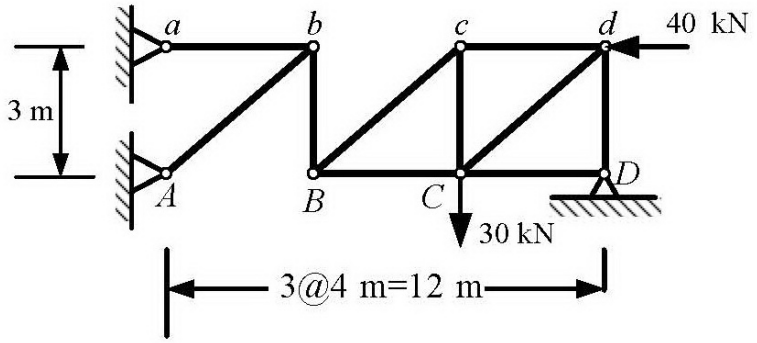


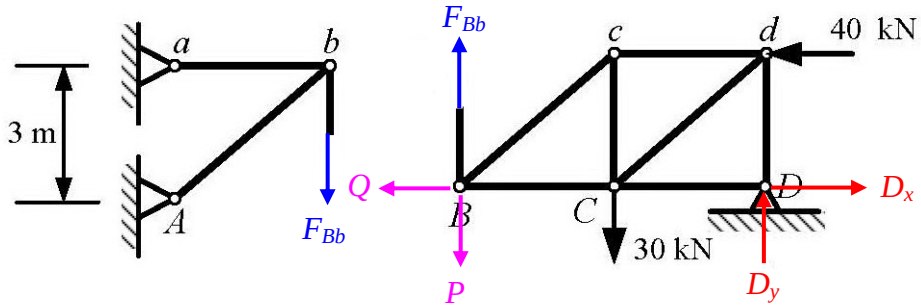
$$\Delta_{Ch} = \frac{1}{3}\cdot 5\cdot\frac{275}{EI}\cdot 4 + \frac{1}{3}\cdot 5\cdot\frac{(-100)}{EI}\cdot 4 + \frac{1}{2}\cdot 5\cdot\frac{(-75)}{EI}\cdot 4$$

$$+\frac{1}{3}\cdot(-5)\cdot\frac{(-250)}{EI}\cdot 5 + \frac{1}{3}\cdot(-3)\cdot\frac{150}{EI}\cdot 3 + \frac{1}{2}\cdot(-2)\cdot\frac{150}{EI}\cdot 3$$

$$= \frac{5500}{3EI} - \frac{2000}{3EI} - \frac{1500}{2EI} + \frac{6250}{3EI} - \frac{450}{EI} - \frac{450}{EI} = \frac{1600}{EI}\,(\leftarrow)$$

6. 給一桁架如下圖所示，假設所有桿件的 $EA=10^5$ kN ，試以卡氏第二定理求解 B 點的水平與垂直位移。 (20%) （107 地特四等)





利用斷面法將問題拆成兩部份，如上所示。

$$\sum F_x = 0 \quad \Rightarrow D_x = 40 + Q$$

$$\sum M_B = 0 \quad \Rightarrow 40 \cdot 3 + D_y \cdot 8 - 30 \cdot 4 = 0 \quad \Rightarrow D_y = 0$$

$$\sum F_y = 0 \quad \Rightarrow F_{Bb} = 30 + P$$

由 D 結點可得 $F_{Dd} = 0$， $F_{CD} = 40 + Q$

由 d 結點可得 $F_{Cd} = 0$， $F_{cd} = -40$

由 c 結點可得 $F_{cB} = -50$， $F_{Cc} = 30$

由 C 結點可得 $F_{BC} = 40 + Q$，

由 B 結點可得 $F_{Bb} = 30 + P$，

由 b 結點可得 $F_{Ab} = -\frac{5}{3}(30 + P)$， $F_{ab} = \frac{4}{3}(30 + P)$

桿件	桿件軸力 N_i	桿長 L_i	$\frac{\partial N_i}{\partial P}$	$\frac{\partial N_i}{\partial Q}$	N_i $(P=0)$ $(Q=0)$	$\frac{\partial N_i}{\partial P}(\frac{N_i L_i}{AE})$	$\frac{\partial N_i}{\partial Q}(\frac{N_i L_i}{AE})$
ab	$\frac{4}{3}(30+P)$	4	$\frac{4}{3}$	0	40	$\frac{640}{3AE}$	0
Ab	$-\frac{5}{3}(30+P)$	5	$-\frac{5}{3}$	0	-50	$\frac{1250}{3AE}$	0
Bb	$30+P$	3	1	0	30	$\frac{90}{AE}$	0
Bc	-50	5	0	0	-50	0	0
BC	$40+Q$	4	0	1	40	0	$\frac{160}{AE}$
Cc	30	3	0	0	30	0	0
CD	$40+Q$	4	0	1	40	0	$\frac{160}{AE}$
Cd	0	5	0	0	0	0	0
cd	-40	4	0	0	-40	0	0
Dd	0	3	0	0	0	0	0

$$\Delta_{By}=\frac{\partial U}{\partial P}=\sum\frac{\partial N_i}{\partial P}(\frac{N_i L_i}{AE})=\frac{640}{3AE}+\frac{1250}{3AE}+\frac{90}{AE}=\frac{720\ (\text{kN}\cdot\text{m})}{AE}$$

$$=0.0072\ (\text{m})=7.2\ (\text{mm})\quad(\downarrow)$$

$$\Delta_{Bx}=\frac{\partial U}{\partial Q}=\sum\frac{\partial N_i}{\partial Q}(\frac{N_i L_i}{AE})=\frac{160}{AE}+\frac{160}{AE}=\frac{320\ (\text{kN}\cdot\text{m})}{AE}$$

$$=0.0032\ (\text{m})=3.2\ (\text{mm})\quad(\leftarrow)$$