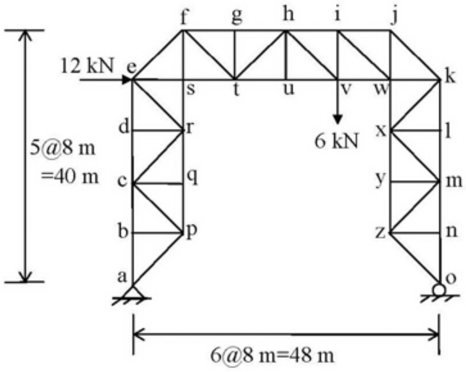
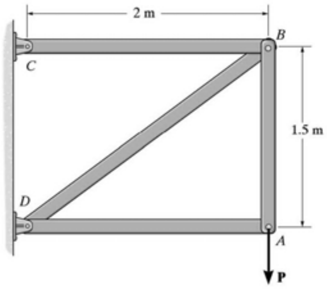
國立臺灣海洋大學河海工程研究所 2026 材料力學 2A 班第三次大考參考解答

日期：2026 年 06 月 11 日　　姓名：_______________　　學號：_______________

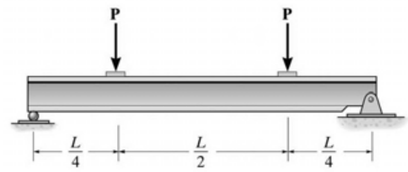
1. 分析如右圖所示桁架結構(各桿件 E、A 均相同)，試問:
   (1) 各支承反力為何? (6%)
   (2) 零力桿件有幾根? (2%)
   (3) 桿件 jk、wk 之受力為何? (6%)
   (4) 最大受力桿件為哪根且大小為何? (6%)



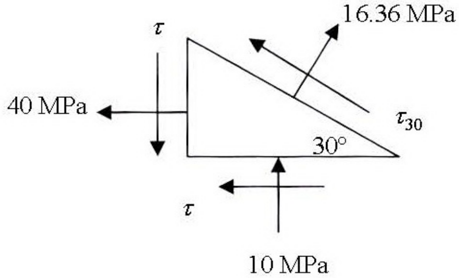
2. 如下圖所示為 A-36 鋼所組成的桁架結構，試求在任何構件均不會產生永久變形下的最大作用力 P 和最大總應變能。(任一構件截面積均為 $2.5\times10^3\ \text{mm}^2$, $E = 200\ \text{GPa}$, $\sigma_Y = 250\ \text{MPa}$) (20%)



3. 試求如下圖所示樑的彎曲應變能。 (20%)

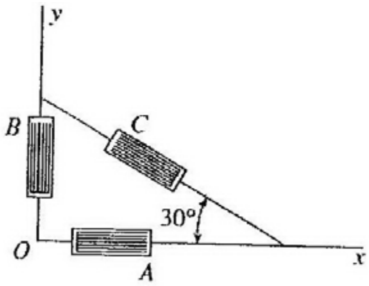


4. 如下圖所示某點之應力狀況，請計算
   (1) 未知應力 τ 與 τ_{30}。 (10%)
   (2) 主應力與其所在平面，並繪出相對應元素圖。 (10%)



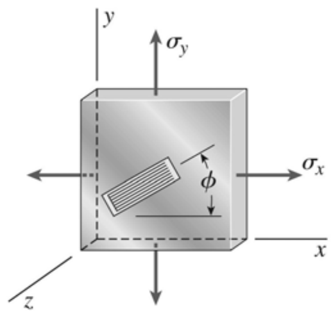
5. 一菊花座應變規貼附於樑上。各量規之讀數分別為$\varepsilon_A = 1200\times10^{-6}$、$\varepsilon_B = 200\times10^{-6}$和$\varepsilon_c = 200\times10^{-6}$。試求:

(1)平面主應變 (7%)　　(2)最大平面剪應變 (7%)　　(3)平均正應變 (6%)



6. 一銅板的彈性模數為$E = 110$ GPa，蒲松比$\nu = 0.34$，受到 x 與 y 的雙軸應力作用(如下圖)。貼在板上的應變規角度為$\phi = 35°$。若$\sigma_x = 74$ (MPa)，量得的應變為$\varepsilon = 390\times10^{-6}$，平面內最大剪應力$(\tau_{\max})_{xy}$與最大剪應變$(\gamma_{\max})_{xy}$是多少？

(20%)



7. (1) 上完了一學期的材料力學，對於這門課在學習上有何心得或感想? (5%)
(2) 對於老師的教學方式或是要如何協助同學們學習好這門課有何建議? (5%)
(兩題請分開寫，有寫才有分)

(參考公式)

平面應力轉換方程式

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

平面應變轉換方程式

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

彎曲公式: $\sigma = -\dfrac{My}{I}$， **剪力公式:** $\tau = \dfrac{VQ}{It}$， **扭轉公式:** $\tau = \dfrac{Tc}{J}$

參考解答:

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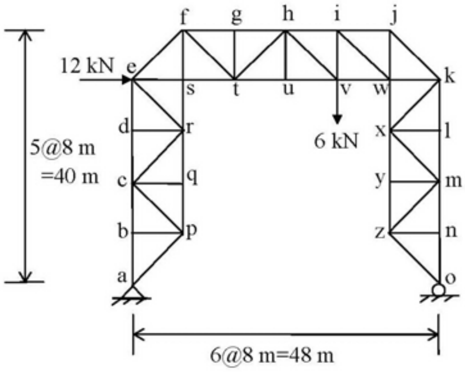


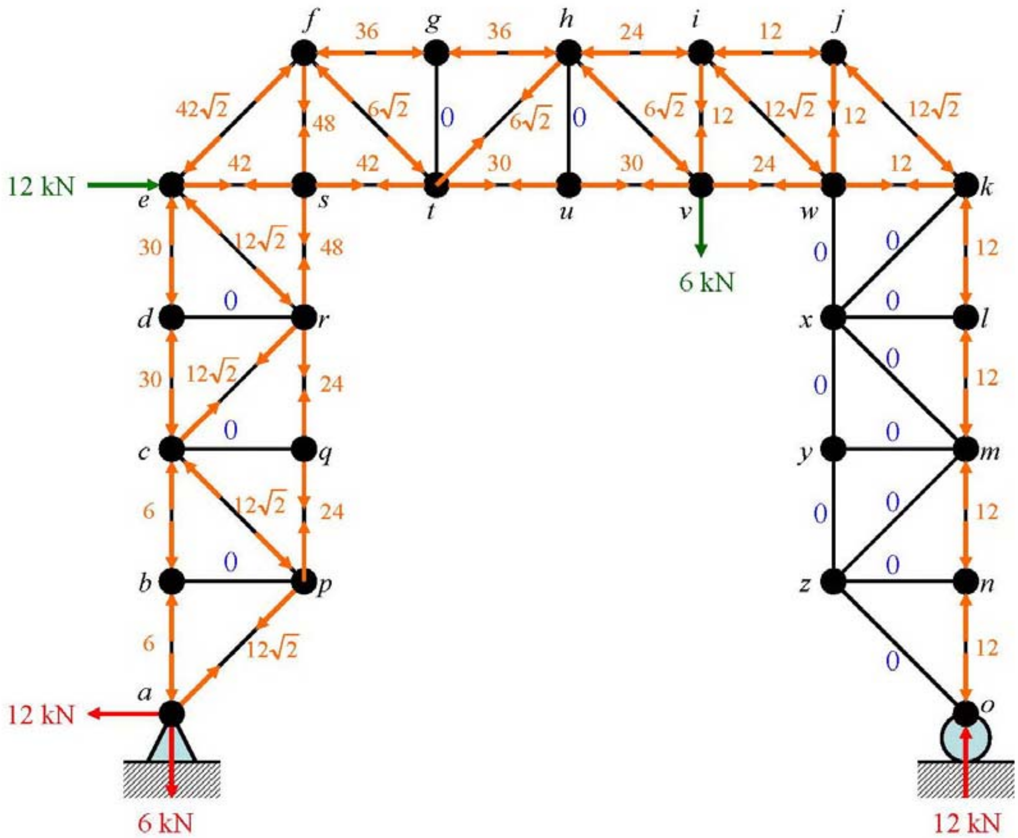
(1) $a_x = 12\,(\text{kN})$ (向左), $a_y = 6\,(\text{kN})$ (向下)

$o_y = 12\,(\text{kN})$ (向上)

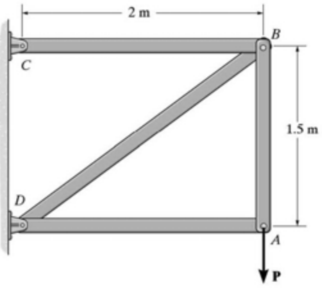
(2) 15 根

(3) $F_{jk} = 12\sqrt{2}\,(\text{kN})$ (壓), $F_{wk} = 12\,(\text{kN})$ (拉)

(4) 最大受力桿件為 ef, $F_{ef} = 42\sqrt{2}\,(\text{kN})$ (壓)



2. 如下圖所示為 A-36 鋼所組成的桁架結構，試求在任何構件均不會產生永久變形下的最大作用力 P 和最大總應變能。（任一構件截面積均為 $2.5\times10^3\ \text{mm}^2$, $E = 200\ \text{GPa}$，$\sigma_Y = 250\ \text{MPa}$）(20%)



$C_x = \dfrac{4}{3}P$ (向左), $C_y = 0$ (向左)

$D_x = \dfrac{4}{3}P$ (向右), $D_y = P$ (向上)

可得 $F_{AB} = P$ (拉), $F_{BD} = \dfrac{5}{3}P$ (壓), $F_{BC} = \dfrac{4}{3}P$ (拉)，$F_{AD} = 0$

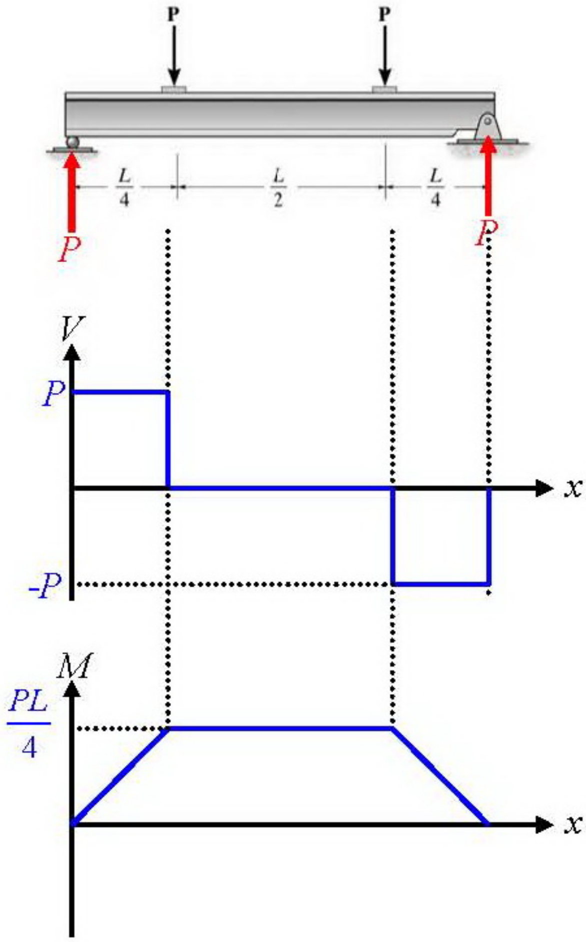
$\therefore$ 最大受力桿件為 BD

$$\sigma_Y = \frac{F_{BD}}{A} \quad \Rightarrow 250\times10^6 = \frac{\frac{5}{3}P}{2.5\times10^{-3}} \quad \Rightarrow P = 375\times10^3\ (\text{N}) = 375\ (\text{kN})$$

$\therefore\ F_{AB} = 375\ (\text{kN})$，$F_{BD} = 625\ (\text{kN})$，$F_{BD} = 500\ (\text{kN})$

$$U_i = \sum\frac{N^2L}{2AE} = \frac{(375^2\times1.5 + 625^2\times2.5 + 500^2\times2)\times10^6}{2\times2.5\times10^{-3}\times200\times10^9} = 1687.5\ (\text{J}) = 1.6875\ (\text{kJ})$$

3. 試求如下圖所示樑的彎曲應變能。 (20%)



$$U_i = \int_0^L \frac{M^2}{2EI}dx = \frac{2}{2EI}[\int_0^{\frac{L}{4}} (Px_1)^2\,dx_1 + \int_{\frac{L}{4}}^{\frac{L}{2}} (\frac{PL}{4})^2\,dx_2]$$

$$= \frac{1}{EI}[\frac{P^2}{3}(\frac{L}{4})^3 + \frac{P^2L^2}{16}(\frac{L}{2} - \frac{L}{4})]$$

$$= \frac{P^2L^3}{48EI}$$

4. 如下圖所示某點之應力狀況，請計算

(1) 未知應力 τ 與 τ_{30}。 (10%)

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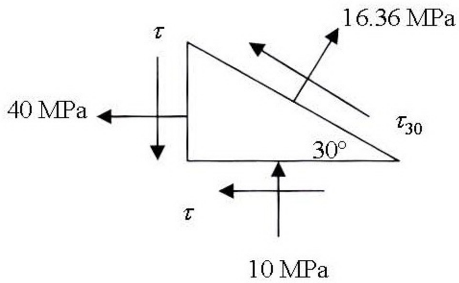
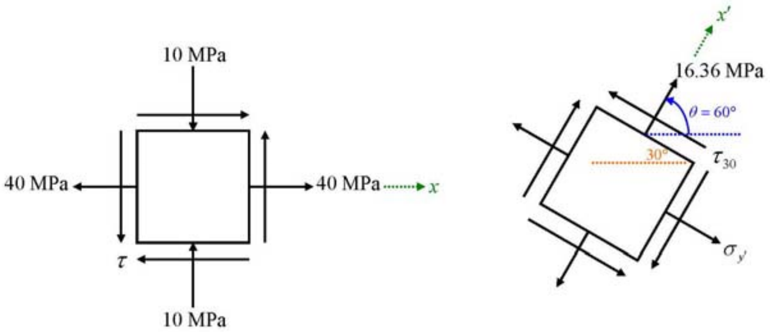




(1) 由圖可知：$\sigma_x = 40\,(\text{MPa})$，$\sigma_y = -10\,(\text{MPa})$，$\tau_{xy} = \tau\,(\text{MPa})$，$\theta = 60°$

$$\sigma_{x'} = 16.36\,(\text{MPa}),\ \tau_{x'y'} = \tau_{30}\,(\text{MPa})$$

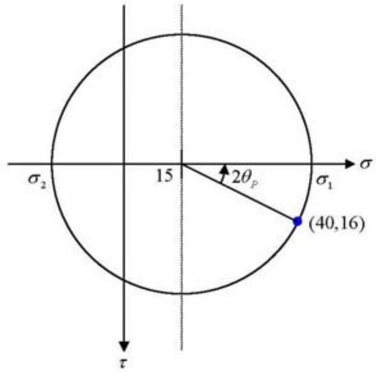
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

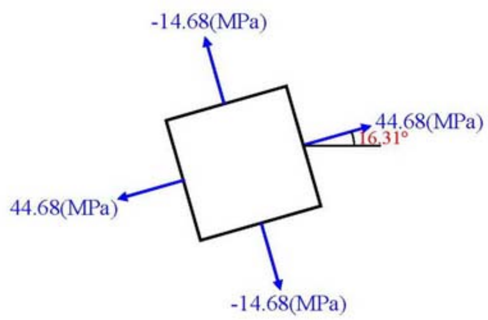
$$\Rightarrow 16.36 = \frac{40 + (-10)}{2} + \frac{40 - (-10)}{2}\cos 120° + \tau \cdot \sin 120° \quad \Rightarrow \tau \approx 16\,(\text{MPa})$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\Rightarrow \quad \tau_{30} = -\frac{40 - (-10)}{2}\sin 120° + 16 \cdot \cos 120° = -29.65\,(\text{MPa})$$

(2) $\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2} = 15$，$O = (\sigma_{avg}, 0) = (15, 0)$，$A = (\sigma_x, \tau_{xy}) = (40, 16)$





$R=\sqrt{(40-15)^2+16^2}=29.68$

$\sigma_1=\sigma_{avg}+R=15+29.68=44.68\,(\text{MPa})$

$\sigma_2=\sigma_{avg}-R=15-29.68=-14.68\,(\text{MPa})$

$\tan 2\theta_P=\dfrac{16}{40-15} \quad \Rightarrow 2\theta_P=\dfrac{16}{40-15}=0.5693\,(\text{rad})=32.62°$

$\Rightarrow \theta_P=16.31°$ (逆)

5. 一菊花座應變規貼附於樑上。各量規之讀數分別為$\varepsilon_A=1200\times10^{-6}$、$\varepsilon_B=200\times10^{-6}$和$\varepsilon_c=200\times10^{-6}$。試求:

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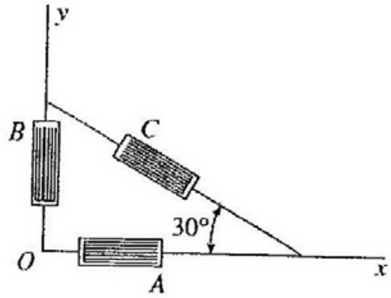

$$\varepsilon_{x'}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta+\frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_A=1200\times10^{-6}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 0°+\frac{\gamma_{xy}}{2}\sin 0° \qquad \Rightarrow \varepsilon_x=1200\times10^{-6}$$

$$\varepsilon_B=200\times10^{-6}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 180°+\frac{\gamma_{xy}}{2}\sin 180° \qquad \Rightarrow \varepsilon_y=200\times10^{-6}$$

$$\varepsilon_C=200\times10^{-6}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 300°+\frac{\gamma_{xy}}{2}\sin 300°$$

$$\Rightarrow \gamma_{xy}=1000\sqrt{3}\times10^{-6}=1732\times10^{-6}$$

(1) $\varepsilon_{1,2}=\dfrac{\varepsilon_x+\varepsilon_y}{2}\pm\sqrt{(\dfrac{\varepsilon_x-\varepsilon_y}{2})^2+(\dfrac{\gamma_{xy}}{2})^2}=1700\times10^{-6}$ and -300×10^{-6}

(2) $\dfrac{\gamma_{\max}}{2}=\sqrt{(\dfrac{\varepsilon_x-\varepsilon_y}{2})^2+(\dfrac{\gamma_{xy}}{2})^2}=1000\times10^{-6} \qquad \Rightarrow \gamma_{\max}=2000\times10^{-6}$

(3) $\varepsilon_{avg}=\dfrac{\varepsilon_1+\varepsilon_2}{2}=700\times10^{-6}$

6. 一銅板的彈性模數為 $E = 110\text{ GPa}$，蒲松比 $\nu = 0.34$，受到 x 與 y 的雙軸應力作用（如下圖）。貼在板上的應變規角度為 $\phi = 35°$。若 $\sigma_x = 74\ (\text{MPa})$，量得的應變為 $\varepsilon = 390 \times 10^{-6}$，平面內最大剪應力 $(\tau_{\max})_{xy}$ 與最大剪應變 $(\gamma_{\max})_{xy}$ 是多少？

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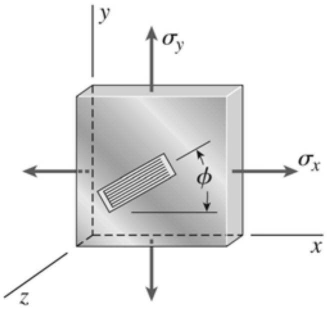

$\sigma_x = 74\text{ MPa}$, $\phi = 35°$, $\varepsilon_{x'} = \varepsilon = 390 \times 10^{-6}$, $\tau_{xy} = 0 \Rightarrow \gamma_{xy} = 0$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y), \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\Rightarrow \quad 390 \times 10^{-6} = \frac{1-\nu}{2E}(\sigma_x + \sigma_y) + \frac{1+\nu}{2E}(\sigma_x - \sigma_y)\cos 70°$$

$$\Rightarrow \quad 390 \times 10^{-6} \cdot 2 \cdot 110 \cdot 10^3 = 0.66 \cdot (74 + \sigma_y) + 1.34 \cdot (74 - \sigma_y) \cdot 0.342$$

$$\Rightarrow \quad 3.05 = 0.66\sigma_y - 0.4583\sigma_y$$

$$\Rightarrow \quad \sigma_y = 15.12\ (\text{MPa})$$

$$(\tau_{\max})_{xy} = \frac{\sigma_x - \sigma_y}{2} = 29.44\ (\text{MPa})$$

$$G = \frac{E}{2(1+\nu)} = 41.045\ (\text{GPa})$$

$$(\gamma_{\max})_{xy} = \frac{(\tau_{\max})_{xy}}{G} = \frac{29.44}{41.045 \cdot 10^3} = 7.17 \cdot 10^{-4}$$