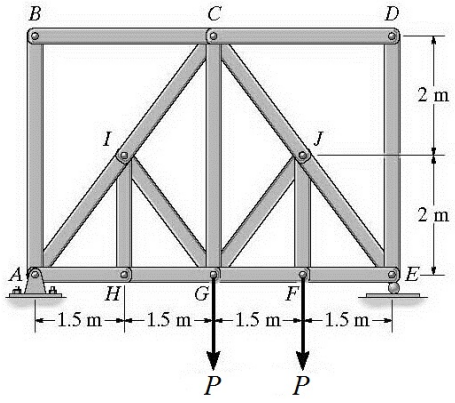
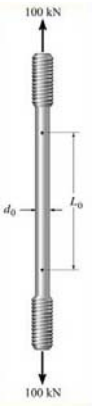
國立臺灣海洋大學河海工程學系 2025 材料力學 2A 班第一次大考參考解答

日期：2025 年 04 月 18 日　　姓名：＿＿＿＿＿＿　　學號：＿＿＿＿＿＿

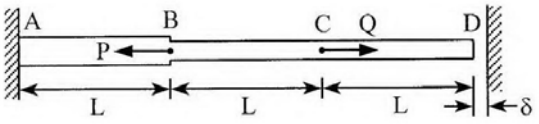
1. 如下左圖所示
   (1) 請指出桁架中何者為零力構件? (6 分)
   (2) 試求桁架中構件 *IC* 與 *CG* 所受之力。(須指明為拉力或壓力) (8 分)
   (3) 若各桿件截面積為 3×10^3 mm^2 且桿件所能承受之最大拉應力或壓應力為 250 MPa ，則 *P* 最大為多少? (6 分)

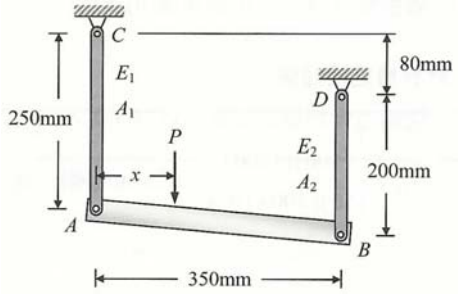




2. 一鋁試片如上右圖所示，其直徑 $d_0 = 25$ mm，標距長度 $L_0 = 250$ mm，若 100 kN 的力造成標距長度拉長 0.73 mm ，試求其彈性模數。並求此力造成試片直徑收縮量。假定 $G_{al} = 26$ GPa ，$\sigma_\gamma = 440$ MPa 。 (20分)

3. 有一ABCD水平桿件如下左圖所示，AB段橫斷面面積 $A_1 = 10\times10^{-4}$ m^2 ，BCD段橫斷面面積 $A_2 = 8\times10^{-4}$ m^2 。ABCD桿件之A點為固定端，D點與牆面有一間隙 $\delta = 2\times10^{-4}$ m 存在。設 L = 2 m ，桿件之彈性係數 E = 200 GPa 。當B點受一集中力 P = 5 kN ，且C點受一集中力 Q = 20 kN 時，試求P及Q作用下，此桿件在A點與D點所受之水平力，並註明反力之方向。(20分)

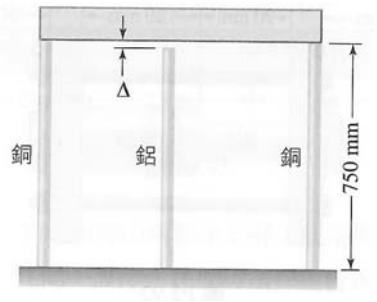


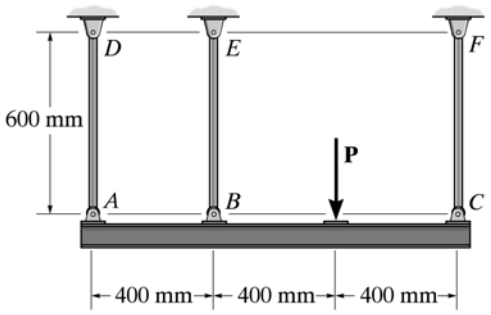


4. 如上右圖所示*AB*剛性桿重量 250 N ，由兩根二力桿件支撐。*AC*桿彈性模數 $E_1 = 3$ MPa 、斷面積 $A_1 = 25$ mm^2 ;*BD*桿彈性模數 $E_2 = 4$ MPa 、斷面積 $A_2 = 20$ mm^2 。若想使原本傾斜的剛性桿受到*P*力作用後變成水平，已知 $P = 180$ N ，試求*P*力作用點與*A*點的距離*x*大小為何? (20分)

5. 水平剛性板與兩根相同的銅桿連結，如下左圖所示。在兩桿中間放置一鋁桿，而鋁桿與板之間有一間隙 $\Delta = 0.18$ mm 。忽略板之質量，且當此組合件的溫度上升85°C 時，試求每根桿件之應力。利用下列數據: (20分)

	A (mm^2)	α (/°C)	E (GPa)
每根銅桿	500	16.8×10^{-6}	120
鋁桿	400	23.1×10^{-6}	70





6. 剛性樑由三根直徑 25 mm 的 A-36 鋼桿支撐。若樑支承一 $P = 230$ kN 的負載，
   (1) 試求每一鋼桿所受的力。 (10 分)
   (2) 若負載 $P = 230$ kN 釋放時，試求每一鋼桿的殘留應力。 (10 分)
   (鋼桿視為完全彈−塑性材料) (A-36 鋼降伏應力 $\sigma_Y = 250$ MPa)

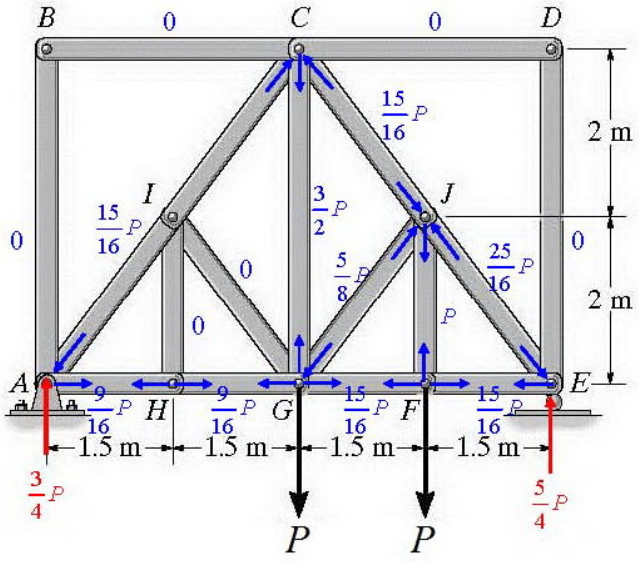
［參考解答］

1. 如下左圖所示

(1) 請指出桁架中何者為零力構件? (6 分)

(2) 試求桁架中構件 *IC* 與 *CG* 所受之力。(須指明為拉力或壓力) (8 分)

(3) 若各桿件截面積為 3×10^{3} mm^{2} 且桿件所能承受之最大拉應力或壓應力為 250 MPa ，則 *P* 最大為多少? (6 分)



(1) 零力桿件 *AB*、*BC*、*CD*、*DE*、*HI*、*GI*

(2) $F_{IC}=\dfrac{15}{16}P$ (壓)

$F_{CG}=\dfrac{3}{2}P$ (拉)

(3) 最大壓力桿件為*EJ*桿， $F_{EJ}=\dfrac{25}{16}P$

$$\sigma=\frac{F_{EJ}}{A}\quad\Rightarrow 250\cdot10^{6}=\frac{\frac{25}{16}P}{3\cdot10^{-3}}\quad\Rightarrow P=480\cdot10^{3}\ (\text{N})=480\ (\text{kN})$$

2. 一鋁試片如下圖所示，其直徑 $d_0 = 25\,\text{mm}$，標距長度 $L_0 = 250\,\text{mm}$，若 $100\,\text{kN}$ 的力造成標距長度拉長 $0.73\,\text{mm}$，試求其彈性模數。並求此力造成試片直徑收縮量。假定 $G_{al} = 26\,\text{GPa}$，$\sigma_\gamma = 440\,\text{MPa}$。(20分) (113結技)

$$\delta = \frac{PL}{AE} \Rightarrow 0.73 = \frac{100\cdot 10^3 \cdot 250}{(\frac{25}{2})^2 \cdot \pi \cdot 10^{-6} \cdot E} \Rightarrow 0.73 = \frac{100\cdot 10^3 \cdot 250}{(\frac{25}{2})^2 \cdot \pi \cdot 10^{-6} \cdot E}$$

$$\Rightarrow E = 69.7666\cdot 10^9\ (\text{Pa}) = 69.7666\ (\text{GPa})$$

$$\sigma = \frac{P}{A} = \frac{100\cdot 10^3}{(\frac{25}{2})^2 \cdot \pi \cdot 10^{-6}} = 203.7183\cdot 10^6\ (\text{Pa}) = 203.7183\ (\text{MPa})$$

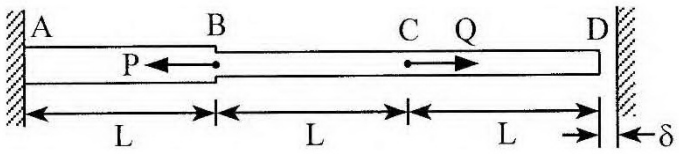
$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{203.7183}{69.7666\cdot 10^3} = 2.9200\cdot 10^{-3}$$

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = \frac{69.7666}{2\cdot 26} - 1 = 0.3417$$

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} \Rightarrow \varepsilon_{\text{lat}} = -\nu \cdot \varepsilon_{\text{long}} = -0.3417 \cdot 2.9200\cdot 10^{-3} = -0.997764\cdot 10^{-3}$$

$$\Delta d = \varepsilon_{\text{long}} \cdot d_0 = -0.997764\cdot 10^{-3} \cdot 25 = -24.9441\cdot 10^{-3}\ (\text{mm})$$

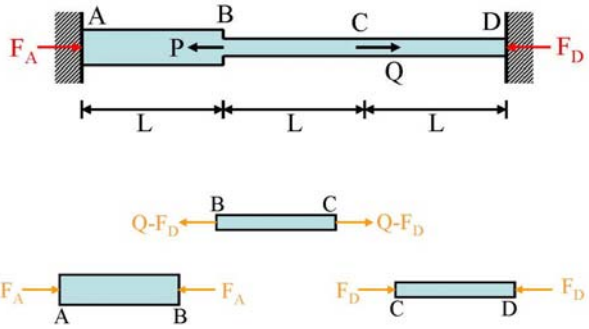
3. 有一ABCD水平桿件如下圖所示，AB段橫斷面面積 $A_1 = 10\times10^{-4}\ m^2$ ，BCD段橫斷面面積 $A_2 = 8\times10^{-4}\ m^2$ 。ABCD桿件之A點為固定端，D點與牆面有一間隙 $\delta = 2\times10^{-4}\ m$ 存在。設 $L = 2\ m$ ，桿件之彈性係數 $E = 200\ GPa$ 。當B點受一集中力 $P = 5\ kN$ ，且C點受一集中力 $Q = 20\ kN$ 時，試求P及Q作用下，此桿件在A點與D點所受之水平力，並註明反力之方向。(20分)　　(113地特三等)



$$\delta_D = \frac{P_1 L}{A_1 E} + \frac{P_2 L}{A_2 E} = \frac{15\cdot10^3\cdot2}{10\cdot10^{-4}\cdot200\cdot10^9} + \frac{20\cdot10^3\cdot2}{8\cdot10^{-4}\cdot200\cdot10^9} = 4\cdot10^{-4}\ (m)$$

$\because\ \delta_D > \delta$

$\therefore$ D點會頂到牆壁



$$\sum F_x = 0 \quad \Rightarrow F_A - P + Q - F_D = 0 \quad \Rightarrow F_A - F_D = -15\cdot10^3 \quad \text{..... (1)}$$

此為一度靜不定，相容方程式為

$$\delta_{D/A} = \delta = 2\cdot10^{-4}\ (m)$$

$$\Rightarrow -\frac{F_A L}{A_1 E} + \frac{(Q - F_D)L}{A_2 E} - \frac{F_D L}{A_2 E} = 2\cdot10^{-4}\ (m)$$

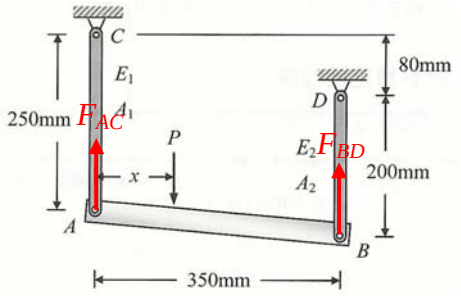
$$\Rightarrow -\frac{F_A\cdot2}{10\cdot10^{-4}\cdot200\cdot10^9} + \frac{(20\cdot10^3 - F_D)\cdot2}{8\cdot10^{-4}\cdot200\cdot10^9} - \frac{F_D\cdot2}{8\cdot10^{-4}\cdot200\cdot10^9} = 2\cdot10^{-4}\ (m)$$

$$\Rightarrow 2F_A + 5F_D = 10\cdot10^3 \quad \text{..... (2)}$$

由(1)與(2)可得 $F_A = -\frac{65}{7}\cdot10^3 = -9.2857\cdot10^3\ (N) = -9.2857\ (kN)$

$$F_D = \frac{40}{7}\cdot10^3 = 5.7143\cdot10^3\ (N) = 5.7143\ (kN)$$

4. 如下圖所示AB剛性桿重量250 N，由兩根二力桿件支撐。AC桿彈性模數$E_1 = 3\ \text{MPa}$、斷面積$A_1 = 25\ \text{mm}^2$；BD桿彈性模數$E_2 = 4\ \text{MPa}$、斷面積$A_2 = 20\ \text{mm}^2$。若想使原本傾斜的剛性桿受到P力作用後變成水平，已知$P = 180\ \text{N}$，試求P力作用點與A點的距離x大小為何? (20分)



$$\sum M_A = 0 \quad \Rightarrow F_{BD}\cdot 350 - P\cdot x - 250\cdot 175 = 0 \quad \Rightarrow F_{BD} = \frac{18}{35}x + 125$$

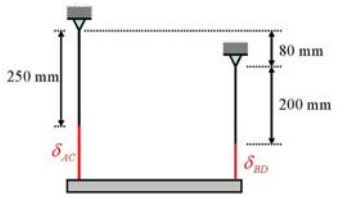
$$\sum F_y = 0 \quad \Rightarrow F_{AC} + F_{BD} - P - 250 = 0 \quad \Rightarrow F_{AC} = 305 - \frac{18}{35}x$$

$$\delta_{AC} = \frac{F_{AC}L_{AC}}{A_1E_1} = \frac{(305 - \frac{18}{35}x)\cdot 250\cdot 10^{-3}}{25\cdot 10^{-6}\cdot 3\cdot 10^{6}} = (\frac{3050}{3} - \frac{12}{7}x)\cdot 10^{-3}\ \text{(m)}$$

$$= \frac{3050}{3} - \frac{12}{1}x\ \text{(mm)}$$

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{A_2E_2} = \frac{(\frac{18}{35}x + 125)\cdot 200\cdot 10^{-3}}{20\cdot 10^{-6}\cdot 4\cdot 10^{6}} = (\frac{9}{7}x + \frac{625}{2})\cdot 10^{-3}\ \text{(m)}$$

$$= \frac{9}{7}x + \frac{625}{2}\ \text{(mm)}$$



相容方程式：$\delta_{AC} + 250 = \delta_{BD} + 80 + 200$

$$\Rightarrow \delta_{AC} - \delta_{BD} = 30$$

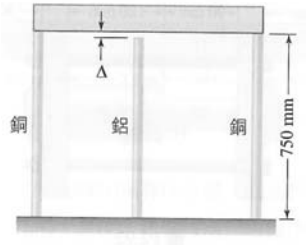
$$\Rightarrow (\frac{3050}{3} - \frac{12}{7}x) - (\frac{9}{7}x + \frac{625}{2}) = 30$$

$$\Rightarrow -3x = 30 - \frac{3050}{3} + \frac{625}{2} = -\frac{4045}{6}$$

$$\Rightarrow x = \frac{4045}{18} = 24.7222\ \text{(mm)}$$

5. 水平剛性板與兩根相同的銅桿連結，如下圖所示。在兩桿中間放置一鋁桿，而鋁桿與板之間有一間隙 $\Delta = 0.18$ mm 。忽略板之質量，且當此組合件的溫度上升85°C 時，試求每根桿件之應力。利用下列數據: (20分)

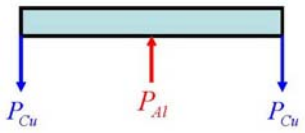
	A (mm^2)	α (/°C)	E (GPa)
每根銅桿	500	16.8×10^{-6}	120
鋁桿	400	23.1×10^{-6}	70

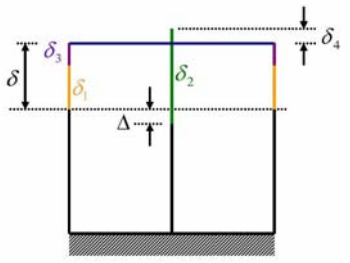


$\delta_1 = \alpha_{cu} \cdot \Delta T \cdot L = 16.8 \cdot 10^{-6} \cdot 85 \cdot 750 = 1.071\,(\text{mm})$

$\delta_2 = \alpha_{Al} \cdot \Delta T \cdot L = 23.1 \cdot 10^{-6} \cdot 85 \cdot 750 = 1.472625\,(\text{mm})$

$\delta_2 - \Delta = 1.292625\,(\text{mm}) > \delta_1$ 表示鋁桿會頂到剛性板





$\sum F_y = 0 \quad \Rightarrow P_{Al} - 2 \cdot P_{cu} = 0 \quad \Rightarrow P_{Al} = 2P_{cu}$

又 $\delta_3 = \dfrac{P_{cu}L}{A_{cu}E_{cu}}$, $\delta_4 = \dfrac{P_{Al}L}{A_{Al}E_{Al}}$

相容方程式: $\delta = \delta_1 + \delta_3 = \delta_2 - \delta_4 - \Delta$

$$\Rightarrow \alpha_{cu} \cdot \Delta T \cdot L + \frac{P_{cu}L}{A_{cu}E_{cu}} = \alpha_{Al} \cdot \Delta T \cdot L - \frac{P_{Al}L}{A_{Al}E_{Al}} - \Delta$$

$$\Rightarrow 1.071 + \frac{P_{cu} \cdot 750}{500 \cdot 10^{-6} \cdot 120 \cdot 10^{9}} = 1.292625 - \frac{P_{Al} \cdot 750}{400 \cdot 10^{-6} \cdot 70 \cdot 10^{9}}$$

$$\Rightarrow \frac{P_{Al}}{28} + \frac{P_{cu}}{60} = 295.5$$

$$\Rightarrow P_{cu} = 295.5 \cdot \frac{420}{37} = 3354.3243\,(\text{N})$$

$$\sigma_{cu} = \frac{P_{cu}}{A_{cu}} = \frac{3354.3243}{500 \cdot 10^{-6}} = 6.7086 \cdot 10^{6}\,(\text{Pa}) = 6.7086\,(\text{MPa})$$

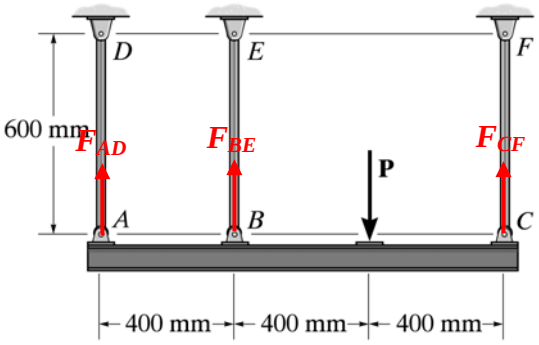
$$\sigma_{Al} = \frac{P_{Al}}{A_{cu}} = \frac{2 \cdot 3354.3243}{400 \cdot 10^{-6}} = 16.7716 \cdot 10^{6}\,(\text{Pa}) = 16.7716\,(\text{MPa})$$

6. 剛性樑由三根直徑 25 mm 的 A-36 鋼桿支撐。若樑支承一 $P = 230$ kN 的負載，

(1) 試求每一鋼桿所受的力。 (10 分)

(2) 若負載 $P = 230$ kN 釋放時，試求每一鋼桿的殘留應力。 (10 分)

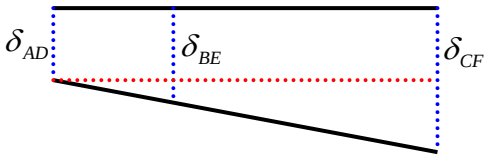
(鋼桿視為完全彈−塑性材料) (A-36 鋼降伏應力 $\sigma_Y = 250$ MPa) (106 結技)



(1)

$$\sum F_y = 0 \quad \Rightarrow F_{AD} + F_{BE} + F_{CF} = P \quad \ldots\ldots \text{(a)}$$

$$\sum M_A = 0 \quad \Rightarrow F_{BE} \cdot 400 + F_{CF} \cdot 1200 - P \cdot 800 = 0 \quad \Rightarrow F_{BE} + 3F_{CF} = 2P \ \ldots\ldots \text{(b)}$$



由幾何位移可看出相容方程為 $\dfrac{\delta_{BE} - \delta_{AD}}{400} = \dfrac{\delta_{CF} - \delta_{AD}}{1200}$

$$\Rightarrow 3\delta_{BE} - 2\delta_{AD} - \delta_{CF} = 0$$

$$\Rightarrow 3\frac{F_{BE} \cdot L}{AE} - 2\frac{F_{AD} \cdot L}{AE} - \frac{F_{CF} \cdot L}{AE} = 0$$

$$\Rightarrow 3F_{BE} - 2F_{AD} - F_{CF} = 0 \ldots\ldots \text{(c)}$$

由解(a)、(b)、(c)聯立方程可得 $F_{BE} = \dfrac{2}{7}P$， $F_{CF} = \dfrac{4}{7}P$， $F_{AD} = \dfrac{1}{7}P$

$\therefore \quad F_{BE} = \dfrac{2}{7} \cdot 230 = 65.71$ (kN)

$$F_{CF} = \frac{4}{7} \cdot 230 = 131.43 \text{ (kN)}$$

$$F_{AD} = \frac{1}{7} \cdot 230 = 32.86 \text{ (kN)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{65.71 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 133.87 \cdot 10^6 \text{ (Pa)} = 133.87 \text{ (MPa)} \ < \ \sigma_Y$$

$$\sigma_{CF} = \frac{F_{CF}}{A} = \frac{131.43 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 267.74 \cdot 10^6 \text{ (Pa)} = 267.74 \text{ (MPa)} \ > \ \sigma_Y$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{32.86 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 66.94 \cdot 10^6 \text{ (Pa)} = 66.94 \text{ (MPa)} < \sigma_Y$$

$\because$ $\sigma_{CF} > \sigma_Y$ 表示 CF 桿已降服，故須力量重分配

$$\therefore \quad F'_{CF} = \sigma_Y \cdot A = 250 \cdot 10^6 \cdot \pi \cdot 12.5^2 \cdot 10^{-6} = 122718.46 \text{ (N)} = 122.72 \text{ (kN)}$$

$$\sum M_A = 0 \quad \Rightarrow F'_{BE} + 3F'_{CF} = 2P \quad \Rightarrow F'_{BE} = 2P - 3F'_{CF} = 91.84 \text{ (kN)}$$

$$\sum F_y = 0 \quad \Rightarrow F'_{AD} + F'_{BE} + F'_{CF} = P \quad \Rightarrow F'_{AD} = P - F'_{CF} - F'_{BE} = 15.44 \text{ (kN)}$$

$$\sigma'_{BE} = \frac{F'_{BE}}{A} = \frac{91.84 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 187.10 \cdot 10^6 \text{ (Pa)} = 187.10 \text{ (MPa)} < \sigma_Y$$

$$\sigma'_{AD} = \frac{F'_{AD}}{A} = \frac{15.44 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 31.45 \cdot 10^6 \text{ (Pa)} = 31.45 \text{ (MPa)} < \sigma_Y$$

(2) $(\sigma_{CF})_r = \sigma'_{CF} - \sigma_{BE} = 250 - 267.74 = -17.74$ (MPa)

$(\sigma_{BE})_r = \sigma'_{BE} - \sigma_{BE} = 187.10 - 133.87 = 53.23$ (MPa)

$(\sigma_{AD})_r = \sigma'_{AD} - \sigma_{AD} = 31.45 - 66.94 = -35.49$ (MPa)