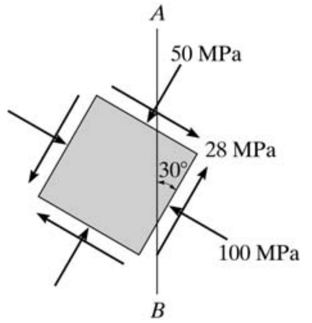
國立臺灣海洋大學河海工程學系 2020 材料力學 2AB 班第三次大考參考解答

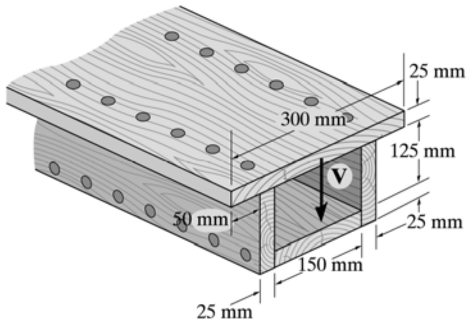
日期：2020 年 06 月 18 日　　姓名：______________　　學號：______________

1. 將構件內一點的應力狀態標示於元素上，如圖一所示。
   (1) 試畫其莫爾圓。 (6%)
   (2) 此點的主應力、最大平面剪應力與其各別對應之方位為何? (8%)
   (3) 作用在 AB 斜面的應力分量為何? (6%)

2. 如圖二，一箱型樑係由四塊平板以具沿樑長各釘距 50 mm 之釘子連接在一起，若各釘子可承受剪力 250 N，試求不破壞釘子可作用在樑上之最大剪力 V。(20%)



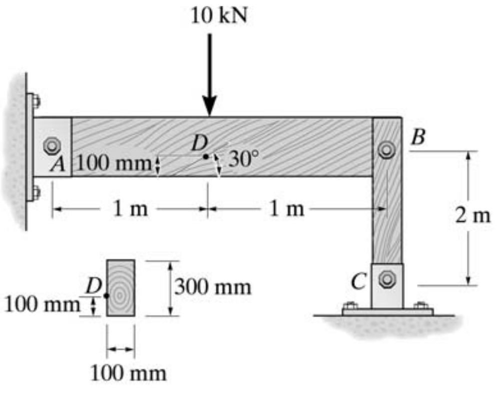
圖一



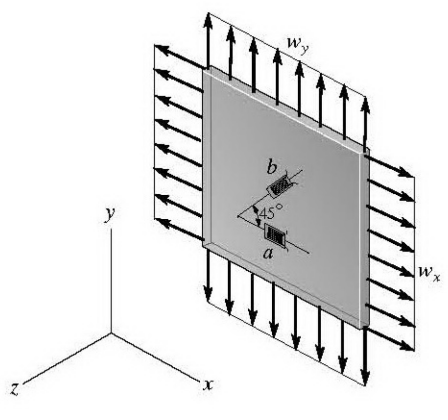
圖二

3. (1) 試求 D 點處分別垂直及平行作用在纖維上的正應力和剪應力。此處纖維如圖三與水平呈30°。D 點恰位於外力 10 kN 的左側。 (10%)
   (2) 試求 D 點處的主應力與其對應之方位。 (10%)

4. 如圖四，兩應變規貼附於一承受均佈負載 $w_x = 700$ kN/m 和 $w_y = -175$ kN/m 的板子表面上。若讀數分別為 $\varepsilon_a = 450\times10^{-6}$ 和 $\varepsilon_b = 100\times10^{-6}$，板厚為 25 mm，試求材料的彈性模數 E、剪力模數 G 和蒲松比 ν。 (20%)



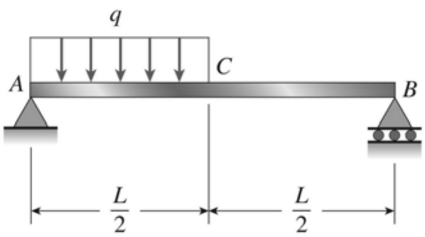
圖三



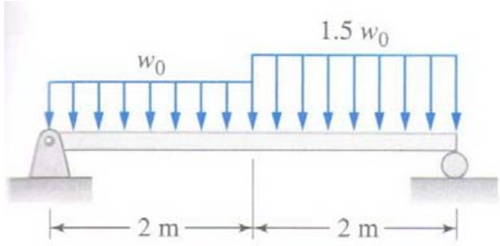
圖四

5. (1) 如圖五，一簡支樑 ACB 於樑的左半段支撐強度為 q 的均佈載重。試以積分法求支承 A 轉角 θ_A 與支承 B 轉角 θ_B 以及 C 點的撓度 v_c。 (12%)

(2) 如圖六，簡支樑的性質 $E = 70$ GPa 及 $I = 30\times10^{-6}$ m^4，欲使跨距中點撓度等於跨距之 $\frac{1}{360}$，求負荷強度 w_0 與樑在 A 端之轉角。 (8%)



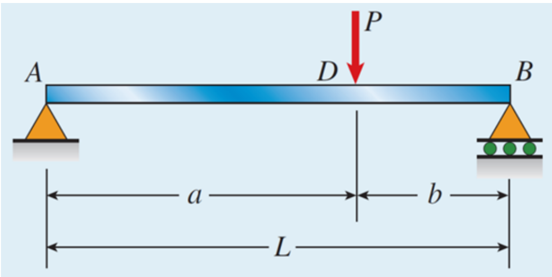
圖五



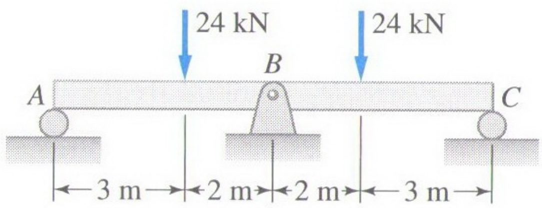
圖六

6. (1) 如圖七，一簡支樑 ADB 受一集中力 P 作用。試以面積−力矩法求支承 A 轉角 θ_A 與支承 B 轉角 θ_B 以及樑的中點 C 的撓度 v_c。 (12%)

(2) 如圖八，樑 ABC 置於三個支承上，求樑在支承 B 點的彎矩與 A 端之轉角。 (8%)



圖七



圖八

(參考公式)

平面應力轉換方程式

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

平面應變轉換方程式

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

彎曲公式： $\sigma = -\frac{My}{I}$，**剪力公式：** $\tau = \frac{VQ}{It}$，**剪力流：** $q = \frac{VQ}{I}$

應變−應力關係式： $\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$，$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

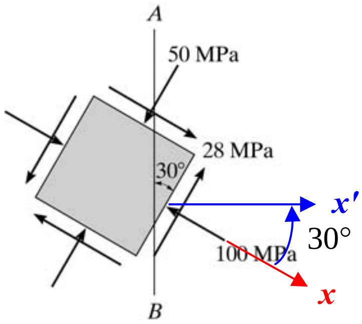
參考解答:

1. 將構件內一點的應力狀態標示於元素上，如下圖一所示。

(1) 試畫其莫爾圓。 (6%)

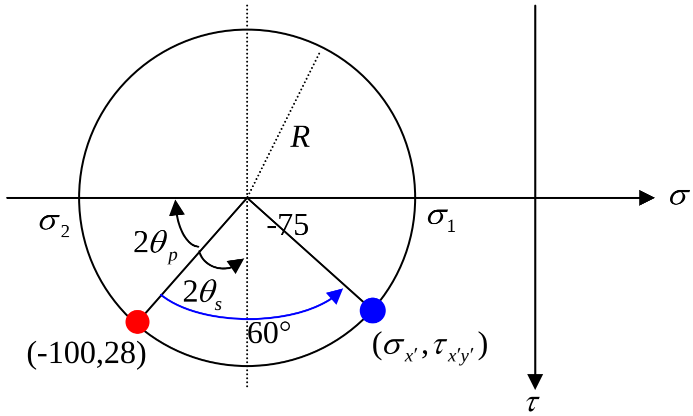
(2) 此點的主應力、最大平面剪應力與其各別對應之方位為何? (8%)

(3) 作用在 *AB* 斜面的應力分量為何? (6%)



(1) $\sigma_x = -100$ MPa，$\sigma_y = -50$ MPa，$\tau_{xy} = 28$ MPa

$$\Rightarrow \sigma_{avg} = \frac{-100-50}{2} = -75\ \text{(MPa)}$$



(2) $R = \sqrt{[-100-(-75)]^2 + 28^2} = 37.54$

主應力: $\sigma_1 = -75 + 37.54 = -37.46$ (MPa)

$\sigma_2 = -75 - 37.54 = -112.54$ (MPa)

$$\tan 2\theta_p = \frac{28}{25} \quad \Rightarrow 2\theta_p = \tan^{-1}\frac{28}{25} = 0.8419\ \text{(rad)} = 48.24^\circ$$

$\Rightarrow \theta_p = 24.12^\circ$ (順時鐘方向旋轉)

最大平面剪應力: $\tau_{\max} = R = 37.54$ (MPa)

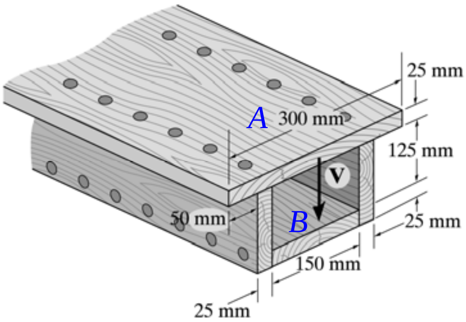
$2\theta_p + 2\theta_s = 90^\circ \quad \Rightarrow \theta_s = 20.88^\circ$ (逆時鐘方向旋轉)

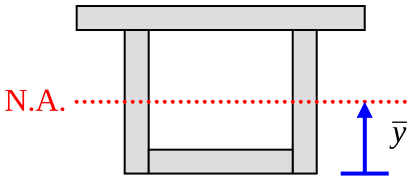
(3) *AB* 斜面為元素逆時鐘方向旋轉 30°，故在莫爾圓上為逆時鐘方向旋轉 60°

$\sigma_{x'} = -75 + 37.54 \cdot \sin(60^\circ - 41.76^\circ) = -63.25$ (MPa)

$\tau_{x'y'} = 37.54 \cdot \cos(60^\circ - 41.76^\circ) = 35.65$ (MPa)

2. 如圖二，一箱型樑係由四塊平板以具沿樑長各釘距 **50 mm** 之釘子連接在一起，若各釘子可承受剪力 **250 N**，試求不破壞釘子可作用在樑上之最大剪力 V。**(20%)**





$$\bar{y}=\frac{\sum y_i A_i}{\sum A_i}=\frac{300\cdot 25\cdot 162.5+(25\cdot 150\cdot 75)\cdot 2+150\cdot 25\cdot 12.5}{300\cdot 25+(25\cdot 150)\cdot 2+150\cdot 25}=97.5\ \ (\text{mm})$$

$$I=\frac{1}{12}300\cdot 25^3+(300\cdot 25)(162.5-97.5)^2+\left[\frac{1}{12}25\cdot 150^3+(25\cdot 150)(97.5-75)^2\right]\cdot 2$$

$$+\frac{1}{12}150\cdot 25^3+(150\cdot 25)(97.5-12.5)^2$$

$$=77.2266\cdot 10^6\ \ (\text{mm}^4)=77.2266\cdot 10^{-6}\ \ (\text{m}^4)$$

若以板 A 上的釘子來設計

$$Q_A=300\cdot 25\cdot (162.5-97.5)=0.4875\cdot 10^6\ \ (\text{mm}^3)=0.4875\cdot 10^{-3}\ \ (\text{m}^3)$$

$$q_A=\frac{1}{2}\frac{VQ_A}{I}=\frac{1}{2}\frac{V\cdot 0.4875\cdot 10^{-3}}{77.2266\cdot 10^{-6}}=3.1563V$$

$$\frac{250}{50\cdot 10^{-3}}=3.1563V\quad \Rightarrow V=1.5841\cdot 10^3\ (\text{N})=1.5841\,(\text{kN})$$

若以板 B 上的釘子來設計

$$Q_B=150\cdot 25\cdot (97.5-12.5)=0.31875\cdot 10^6\ \ (\text{mm}^3)=0.31875\cdot 10^{-3}\ \ (\text{m}^3)$$

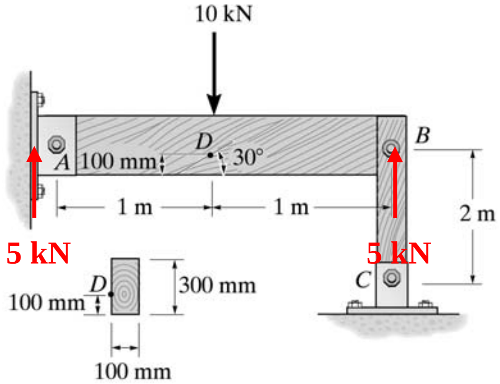
$$q_B=\frac{1}{2}\frac{VQ_B}{I}=\frac{1}{2}\frac{V\cdot 0.31875\cdot 10^{-3}}{77.2266\cdot 10^{-6}}=2.0637V$$

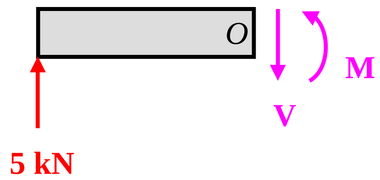
$$\frac{250}{50\cdot 10^{-3}}=2.0637V\quad \Rightarrow V=2.4228\cdot 10^3\ (\text{N})=2.4228\,(\text{kN})$$

$\therefore$ 可知最大的剪力為 $1.5841\,(\text{kN})$

3. (1) 試求 D 點處分別垂直及平行作用在纖維上的正應力和剪應力。此處纖維如圖三與水平呈 $30°$。D 點恰位於外力 10 kN 的左側。 (10%)

(2) 試求 D 點處的主應力與其對應之方位。 (10%)





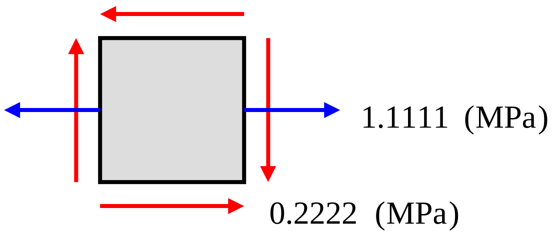
$$\sum F_x = 0 \quad \Rightarrow V = 5 \;\; \text{(kN)} \qquad \text{又} \qquad \sum M_O = 0 \quad \Rightarrow M = 5 \;\; \text{(kN}\cdot\text{m)}$$

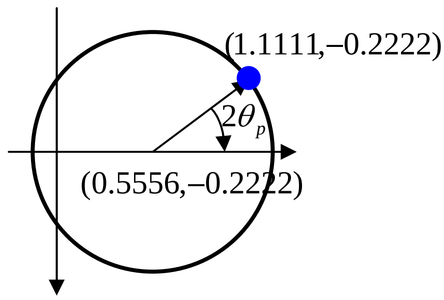
$$I = \frac{1}{12} 100 \cdot 300^3 = 225 \cdot 10^6 \;\; (\text{mm}^4) = 225 \cdot 10^{-6} \;\; (\text{m}^4)$$

$$Q_D = \bar{y}'A' = 100 \cdot 100 \cdot (50 + 50) = 10^6 \;\; (\text{mm}^3) = 10^{-3} \;\; (\text{m}^3)$$

$$\sigma_D = -\frac{My}{I} = -\frac{5 \cdot 10^3 \cdot (-50 \cdot 10^{-3})}{225 \cdot 10^{-6}} = 1.1111 \cdot 10^6 \;\; (\text{Pa}) = 1.1111 \;\; (\text{MPa})$$

$$\tau_D = \frac{VQ}{It} = -\frac{5 \cdot 10^3 \cdot 10^{-3}}{225 \cdot 10^{-6} \cdot 100 \cdot 10^{-3}} = 0.2222 \cdot 10^6 \;\; (\text{Pa}) = 0.2222 \;\; (\text{MPa})$$





由圖可看出 $\sigma_x = 1.1111$ (MPa)，$\sigma_y = 0$ (MPa)，$\tau_{xy} = -0.2222$ (MPa)

$$\sigma_{avg} = \frac{1.1111 + 0}{2} = 0.5556 \;\; (\text{MPa})$$

圓心 $O(0.5556, 0)$，正 x 軸面應力狀態 $A(1.1111, -0.2222)$

$$\therefore \quad R = \sqrt{(1.1111 - 0.5556)^2 + (-0.2222)^2} = 0.5984 \;\; (\text{MPa})$$

主應力：$\sigma_1 = 0.5556 + 0.5984 = 1.1540$ (MPa)

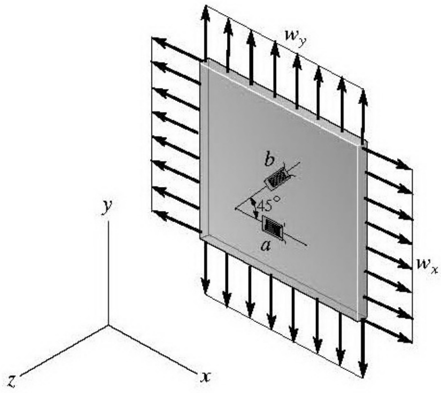
$\sigma_2 = 0.5556 - 0.5984 = -0.0428$ (MPa)

$$\tan(2\theta_p) = \frac{0.2222}{1.1111 - 0.5556} = 0.4 \quad \Rightarrow 2\theta_p = 0.3805 \text{ (rad)} = 21.80°$$

$$\Rightarrow \theta_p = 10.90°$$

$\because$ 為順時針方向轉動 $\quad \therefore \theta_p = -10.90°$

4. 如圖四，兩應變規貼附於一承受均佈負載$w_x = 700$ kN/m和$w_y = -175$ kN/m的板子表面上。若讀數分別為$\varepsilon_a = 450\times10^{-6}$和$\varepsilon_b = 100\times10^{-6}$，試求材料的彈性模數$E$、剪力模數$G$和蒲松比$\nu$。 **(20%)**



$$\sigma_x = \frac{w_x}{t} = \frac{700\cdot10^3}{25\cdot10^{-3}} = 28\cdot10^6 \text{ (Pa)} = 28 \text{ (MPa)}$$

$$\sigma_y = \frac{w_y}{t} = \frac{-175\cdot10^3}{25\cdot10^{-3}} = -7\cdot10^6 \text{ (Pa)} = -7 \text{ (MPa)}$$

$$\sigma_z = 0 \text{ (MPa)}$$

∵ 只受軸力作用，而無剪應力作用

∴ $\gamma_{xy} = 0$

$$\varepsilon_a = 450\times10^{-6} = \varepsilon_x\cdot\cos^2 0° + \varepsilon_y\cdot\sin^2 0° + \gamma_{xy}\sin 0°\cdot\cos 0°$$

$$\Rightarrow \varepsilon_x = 450\times10^{-6}$$

$$\varepsilon_b = 100\times10^{-6} = \varepsilon_x\cdot\cos^2 45° + \varepsilon_y\cdot\sin^2 45° + \gamma_{xy}\sin 45°\cdot\cos 45°$$

$$\Rightarrow \varepsilon_y = -250\times10^{-6}$$

由廣義虎克定律可知:

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)] \quad \Rightarrow 450\cdot10^{-6} = \frac{1}{E}[28-\nu(-7+0)]\cdot10^6 \quad \ldots(1)$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x+\sigma_z)] \quad \Rightarrow -250\cdot10^{-6} = \frac{1}{E}[-7-\nu(28+0)]\cdot10^6 \quad \ldots(2)$$

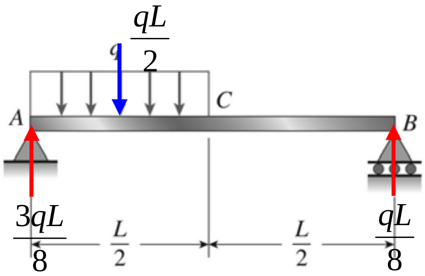
由(1)與(2)可得$\dfrac{7+28\nu}{250} = \dfrac{28+7\nu}{450} \quad \Rightarrow \nu = 0.3548$

代入(1)可得$E = 67.74\cdot10^9$ (Pa) $= 67.74$ (GPa)

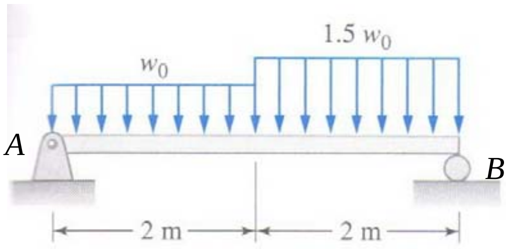
$$G = \frac{E}{2(1+\nu)} = \frac{67.74}{2(1+0.3548)} = 25 \text{ (GPa)}$$

5. **(1)** 如圖五，一簡支樑 ACB 於樑的左半段支撐強度為 q 的均佈載重。試以積分法求支承 A 轉角 θ_A 與支承 B 轉角 θ_B 以及 C 點的撓度 v_c。 **(12%)**

(2) 如圖六，簡支樑的性質 $E = 70$ GPa 及 $I = 30\times10^{-6}$ m^4，欲使跨距中點撓度等於跨距之 $\dfrac{1}{360}$，求負荷強度 w_0 與樑在 A 端之轉角。 **(8%)**

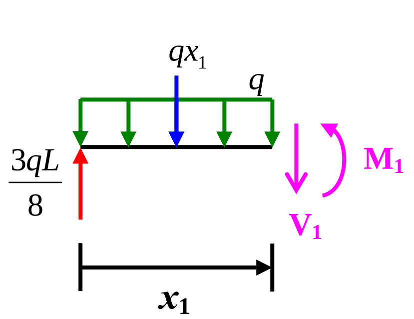


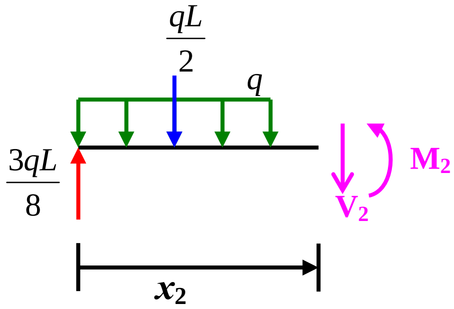
圖五



圖六

(1)





當 $0 \le x_1 < \dfrac{L}{2}$

$$\sum M = 0 \quad \Rightarrow M_1 + qx_1 \cdot \frac{x_1}{2} - \frac{3qL}{8}\cdot x_1 = 0 \quad \Rightarrow M_1 = \frac{3qL}{8}x_1 - \frac{q}{2}x_1^2$$

$$EI\frac{d^2v_1}{dx_1^2} = M_1 = \frac{3qL}{8}x_1 - \frac{q}{2}x_1^2 \quad \Rightarrow EI\frac{dv_1}{dx_1} = \frac{3qL}{16}x_1^2 - \frac{q}{6}x_1^3 + C_1$$

$$\Rightarrow EIv_1 = \frac{qL}{16}x_1^3 - \frac{q}{24}x_1^4 + C_1x_1 + C_2$$

當 $\dfrac{L}{2} < x_2 < L$

$$\sum M = 0 \quad \Rightarrow M_2 + \frac{qL}{2}(x_2 - \frac{L}{4}) - \frac{3qL}{8}\cdot x_2 = 0 \quad \Rightarrow M_2 = \frac{qL^2}{8} - \frac{qL}{8}x_2$$

$$EI\frac{d^2v_2}{dx_2^2} = M_2 = \frac{qL^2}{8} - \frac{qL}{8}x_2 \quad \Rightarrow EI\frac{dv_2}{dx_2} = \frac{qL^2}{8}x_2 - \frac{qL}{16}x_2^2 + C_3$$

$$\Rightarrow EIv_2 = \frac{qL^2}{16}x_2^2 - \frac{qL}{48}x_2^3 + C_3x_2 + C_4$$

由邊界條件可得

$v_1(0) = 0 \quad \Rightarrow C_2 = 0$

$v_2(L) = 0 \quad \Rightarrow C_3L + C_4 = -\dfrac{qL^4}{24} \quad \ldots(1)$

由連續條件可得

$$v_1(\frac{L}{2})=v_2(\frac{L}{2}) \quad \Rightarrow \frac{L}{2}C_1-\frac{L}{2}C_3-C_4=\frac{qL^4}{128} \quad \ldots(2)$$

$$\theta_1(\frac{L}{2})=\theta_2(\frac{L}{2}) \quad \Rightarrow C_1-C_3=\frac{qL^3}{48} \quad \ldots(3)$$

將(3)代入(2)可得 $C_4=\frac{qL^4}{384}$

代入(1)可得 $C_3=-\frac{17qL^3}{384}$

入(3)可得 $C_1=-\frac{3qL^3}{128}$

當 $0\le x_1<\frac{L}{2}$

$$EI\frac{dv_1}{dx_1}=\frac{3qL}{16}x_1^2-\frac{q}{6}x_1^3-\frac{3qL^3}{128}=\frac{q}{384}(72Lx_1^2-64x_1^3-9L^3)$$

$$EIv_1=\frac{qL}{16}x_1^3-\frac{q}{24}x_1^4-\frac{3qL^3}{128}x_1=\frac{qx_1}{384}(24Lx_1^2-16x_1^3-9L^3)$$

當 $\frac{L}{2}<x_2<L$

$$EI\frac{dv_2}{dx_2}=\frac{qL^2}{8}x_2-\frac{qL}{16}x_2^2-\frac{17qL^3}{384}=\frac{qL}{384}(48Lx_2-24x_2^2-17L^2)$$

$$EIv_2=\frac{qL^2}{16}x_2^2-\frac{qL}{48}x_2^3-\frac{17qL^3}{384}x_2+\frac{qL^4}{384}=\frac{qL}{384}(24Lx_2^2-8x_2^3-17L^2x_2+L^3)$$

$$\therefore \quad \theta_A=\theta_1(0)=-\frac{9qL^3}{384EI}=-\frac{3qL^3}{128EI}$$

$$\theta_B=\theta_2(L)=\frac{7qL^3}{384EI}$$

$$v_c=v_1(\frac{L}{2})=-\frac{5qL^4}{768EI}$$

(2) 由重疊法可得

$$v(\frac{L}{2})=-\frac{5w_0L^4}{768EI}+(-\frac{7.5w_0L^4}{768EI})=-\frac{25w_0L^4}{1536EI}$$

$$\Rightarrow -\frac{25w_0\cdot 4^4}{1536\cdot 70\cdot 10^9\cdot 30\cdot 10^{-6}}=-\frac{4}{360}$$

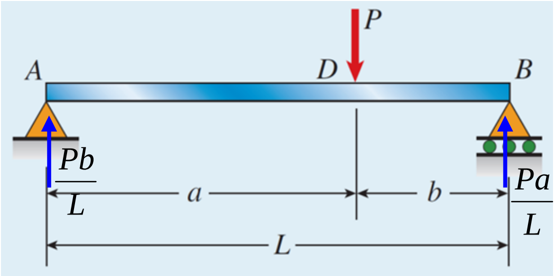
$$\Rightarrow w_0=5600 \quad (\frac{\mathrm{N}}{\mathrm{m}})$$

$$\theta_A=-\frac{3w_0L^3}{128EI}+(-\frac{7\cdot(1.5w_0)L^3}{384EI})=-\frac{19.5\,w_0L^3}{384EI}=-\frac{19.5\cdot 5600\cdot 4^3}{384\cdot 70\cdot 10^9\cdot 30\cdot 10^{-6}}$$

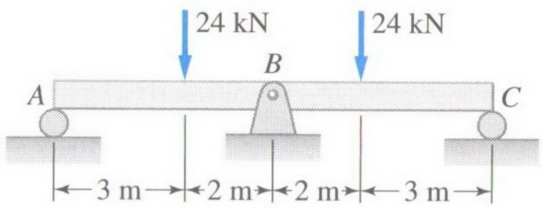
$$=-0.00867\ (\text{rad})=0.50^\circ$$

6. (1) 如圖七，一簡支樑 *ADB* 受一集中力 *P* 作用。試以面積-力矩法求支承 *A* 轉角 θ_A 與支承 *B* 轉角 θ_B 以及樑的中點 *C* 的撓度 v_c。 **(12%)**

(2) 如圖八，樑 *ABC* 置於三個支承上，求樑在支承 *B* 點的彎矩與 *A* 端之轉角。 **(8%)**

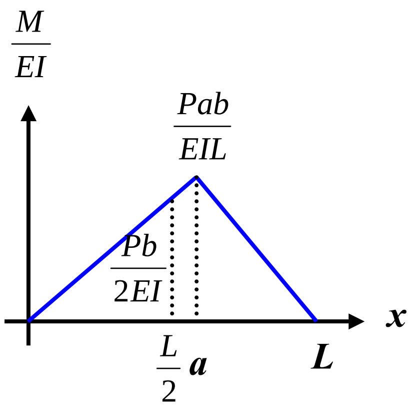


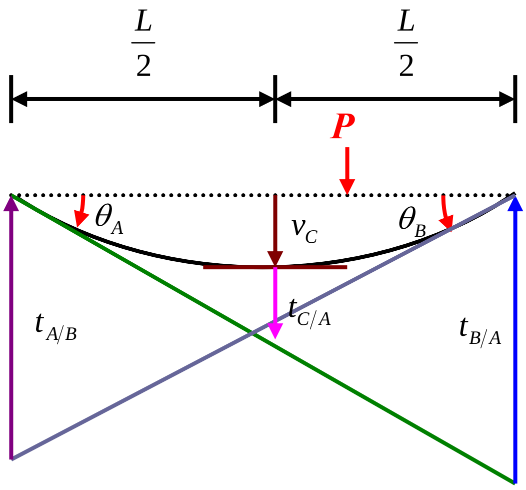
圖七



圖八

(1)





$$t_{B/A}=\frac{1}{2}\cdot a\cdot\frac{Pab}{EIL}\cdot(b+\frac{1}{3}a)+\frac{1}{2}\cdot b\cdot\frac{Pab}{EIL}\cdot\frac{2}{3}b=\frac{1}{6}\cdot\frac{Pab}{EIL}\cdot(a^2+3ab+2b^2)$$

$$=\frac{1}{6}\cdot\frac{Pab}{EI}\cdot(L+b)$$

$$t_{A/B}=\frac{1}{2}\cdot a\cdot\frac{Pab}{EIL}\cdot\frac{2}{3}a+\frac{1}{2}\cdot b\cdot\frac{Pab}{EIL}\cdot(a+\frac{1}{3}b)=\frac{1}{6}\cdot\frac{Pab}{EIL}\cdot(2a^2+3ab+b^2)$$

$$=\frac{1}{6}\cdot\frac{Pab}{EI}\cdot(L+a)$$

$$\theta_A=\frac{t_{B/A}}{L}=\frac{1}{6}\cdot\frac{Pab}{EIL}\cdot(L+b)$$

$\because$ θ_A 為順時鐘方向

$$\therefore\ \theta_A=-\frac{1}{6}\cdot\frac{Pab}{EIL}\cdot(L+b)$$

$$\theta_B=\frac{t_{A/B}}{L}=\frac{1}{6}\cdot\frac{Pab}{EIL}\cdot(L+a)$$

$$t_{C/A}=\frac{1}{2}\cdot\frac{L}{2}\cdot\frac{Pb}{2EI}\cdot\frac{L}{6}=\frac{1}{48}\cdot\frac{PL^2b}{EI}$$

由相似三角形可得 $v_c + t_{C/A} = \frac{1}{2} t_{B/A}$

$$\Rightarrow v_c = \frac{1}{12} \cdot \frac{Pab}{EI} \cdot (L+b) - \frac{1}{48} \cdot \frac{PL^2 b}{EI}$$

$$= \frac{Pb}{48EI} \cdot (4aL + 4ab - L^2)$$

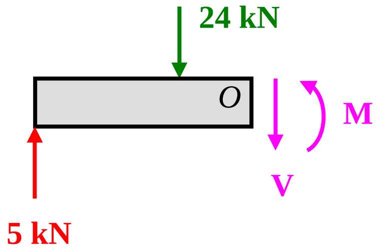
$$= \frac{Pb}{48EI} \cdot (3L^2 - 4b^2)$$

(2) 由重疊法可得

$$v_B = 0 \quad \Rightarrow \frac{24 \cdot 3}{48EI} \cdot (3 \cdot 10^2 - 4 \cdot 3^2) \cdot 2 - \frac{B_y \cdot 5}{48EI} \cdot (3 \cdot 10^2 - 4 \cdot 5^2) = 0$$

$$\Rightarrow B_y \cong 38 \ \ (\text{kN})$$

$$\therefore \ A_y = C_y = \frac{48-38}{2} = 5 \ \ (\text{kN})$$



$$\sum M_O = 0 \ \ M - 24 \cdot 2 - 5 \cdot 5 = 0 \quad \Rightarrow M = -23 \ \ (\text{kN} \cdot \text{m})$$

$$\theta_A = -\frac{1}{6} \cdot \frac{Pab}{EIL} \cdot (L+a) - \frac{1}{6} \cdot \frac{Pab}{EIL} \cdot (L+b) + \frac{1}{6} \cdot \frac{B_y \cdot \frac{L}{2} \cdot \frac{L}{2}}{EIL} \cdot (L + \frac{L}{2})$$

$$= -\frac{1}{6} \cdot \frac{24 \cdot 7 \cdot 3}{EI \cdot 10} \cdot (10+7) - \frac{1}{6} \cdot \frac{24 \cdot 7 \cdot 3}{EI \cdot 10} \cdot (10+3) + \frac{1}{6} \cdot \frac{38 \cdot 5}{2EI} \cdot (10+5)$$

$$= -\frac{29}{2EI}$$