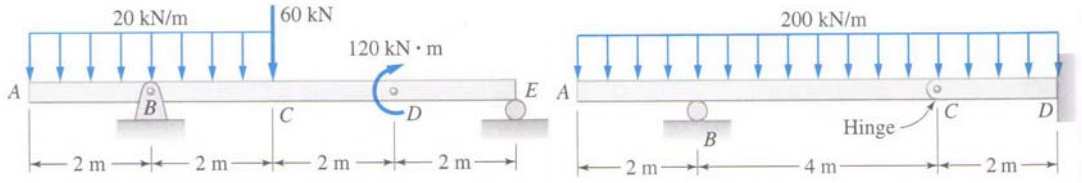
國立臺灣海洋大學河海工程學系 2020 材料力學 2AB 班第二次大考參考解答

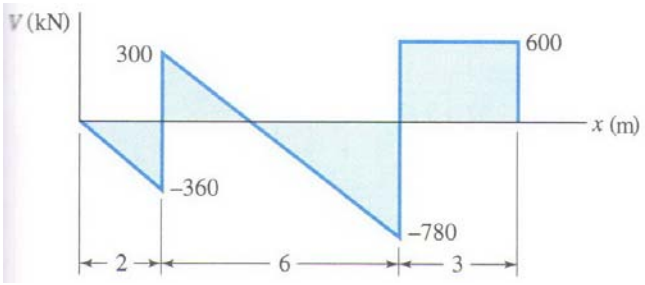
日期：2020 年 05 月 28 日　　姓名：＿＿＿＿＿＿　　學號：＿＿＿＿＿＿

1. 請畫出下面各樑的的剪力圖和彎矩圖。 (20%)

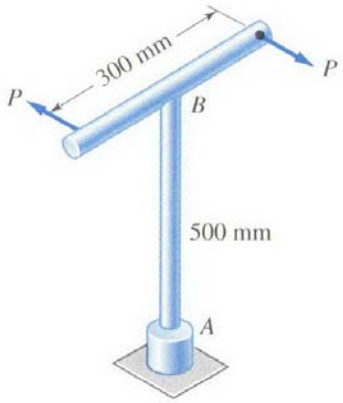


(a)　　(b)

2. 依照所給的剪力圖，繪製相對應的負荷圖與彎矩圖。假設並無力偶作用於梁上。 (10%)



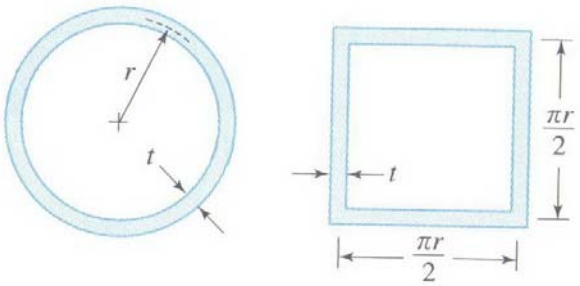
3. 大小為 P 的二力作用於下圖所示的鈑手上。已知鋼軸 AB 的直徑是15 mm。如軸內之剪應力不可超過120 MPa 且扭轉角不可超過5°。試求最大容許 P 值。(鋼的 $G = 80$ GPa) (15%)



4. 兩相同的金屬平版分別被製成如下圖所示之圓形及方形橫截面管件。若兩支管件都受到相同的扭矩，試求:

(1) 剪應力 $\tau_{\text{圓形}}/\tau_{\text{方形}}$ 之比值。 (10%)

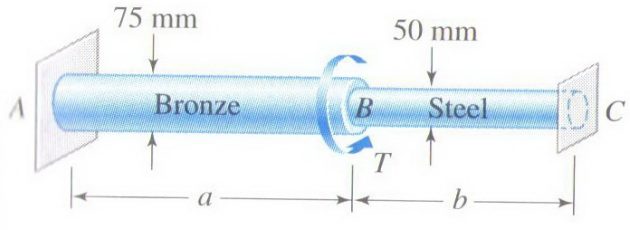
(2) 扭轉角 $\phi_{\text{圓形}}/\phi_{\text{方形}}$ 之比值。 (10%)



5. 如下圖所示，複合軸兩端固定於剛性壁。其中青銅段 AB 直徑為 75 mm 及 $G = 35$ GPa 。而鋼段 BC 的直徑為 50 mm 及 $G = 83$ GPa 。

(1) 已知 $a = 2$ m 和 $b = 1.5$ m 。若青銅(Bronze)與鋼(Steel)之最大剪應力分別不超過 60 MPa 和 80 MPa ，試求所能施加之最大扭矩 T。 (10%)

(2) 若使各材料的應力達其容許極限，試問扭矩 T 和 b/a 應為多少？ (10%)



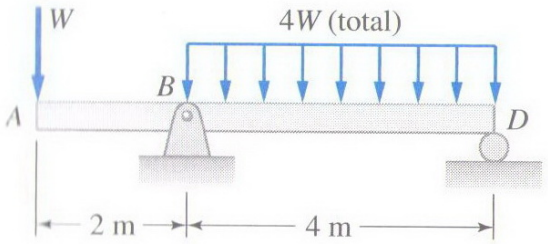
6. 樑承受集中負荷 W 和總合力 $4W$ 之均佈負荷如下左圖，其斷面如下右圖所示。

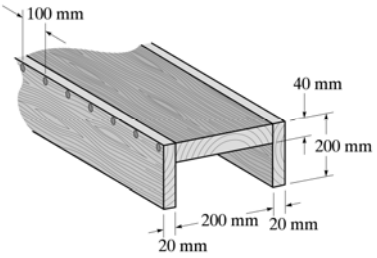
(1) 此樑斷面的中性軸位置？ (4%)

(2) 此斷面中性軸的面積慣性矩 I = ？ (4%)

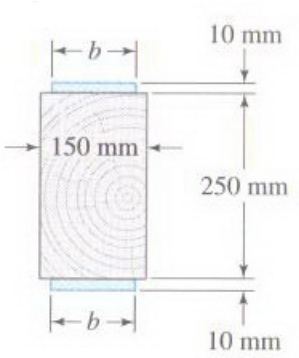
(3) 此樑的最大拉應力和最大壓應力分別為何？ (8%)

(4) 若容許拉伸應力為 60 MPa ，容許壓縮應力為 100 MPa ，試求 W 之最大容許值。 (4%)





7. 木樑(wood)在底和頂端以寬度 b 的鋼板(steel)強化。若木材的容許應力為 10 MPa ，鋼為 120 MPa 且彈性模數比為 $E_{st}/E_{wd} = 15$，試求能承受 40 kN·m 彎矩的最小寬度 b。 (15%)



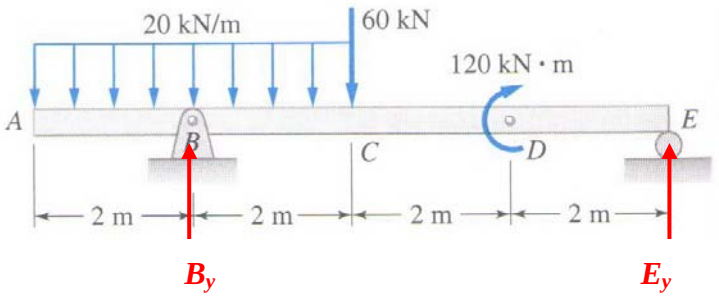
薄壁樑扭轉公式： $\tau_{avg} = \dfrac{T}{2A_m}$

薄壁樑扭轉角公式： $\phi = \dfrac{TL}{4A_m^2 G}\oint \dfrac{ds}{t}$

參考解答:

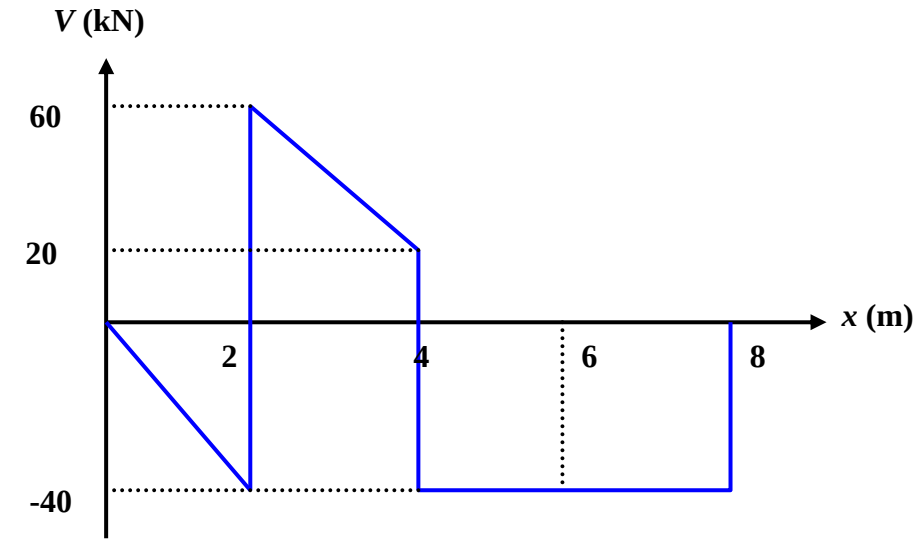
1. 請畫出下面各樑的的剪力圖和彎矩圖。 (20%)

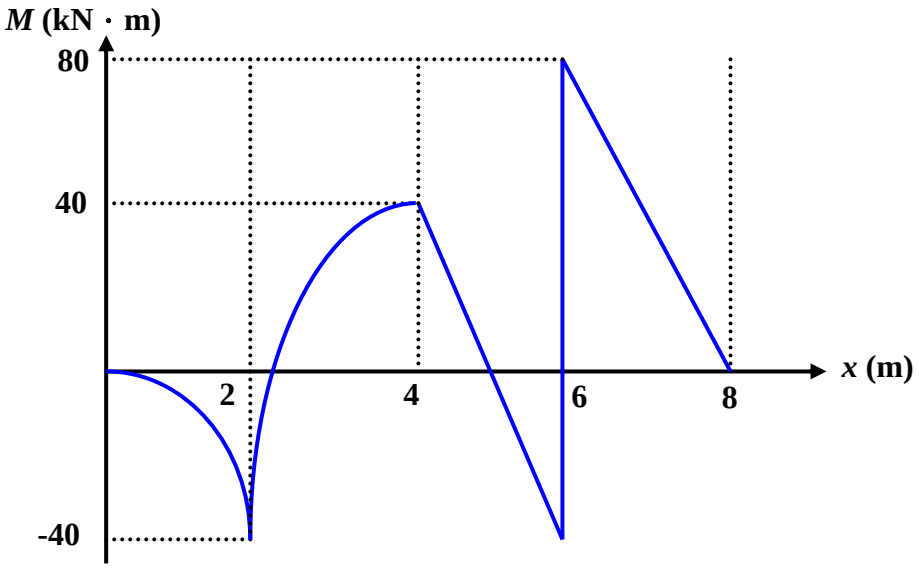
(a)



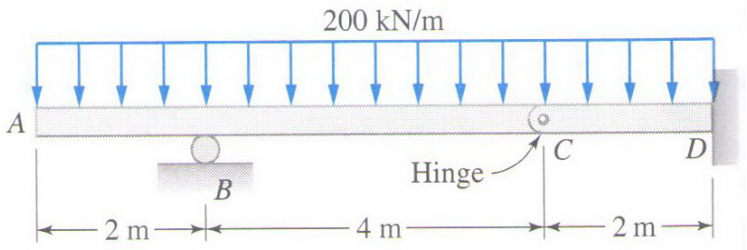
$$\sum M_E = 0 \quad \Rightarrow B_y \cdot 6 - (20 \cdot 4) \cdot 6 - 60 \cdot 4 + 120 = 0 \quad \Rightarrow B_y = 100 \ \text{(kN)}$$

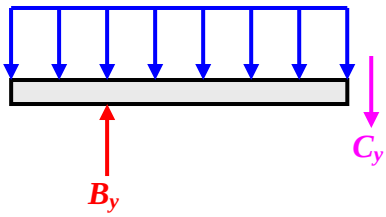
$$\sum F_y = 0 \quad \Rightarrow B_y + E_y - 20 \cdot 4 - 60 = 0 \quad \Rightarrow E_y = 40 \ \text{(kN)}$$





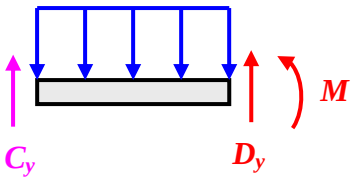
(b)



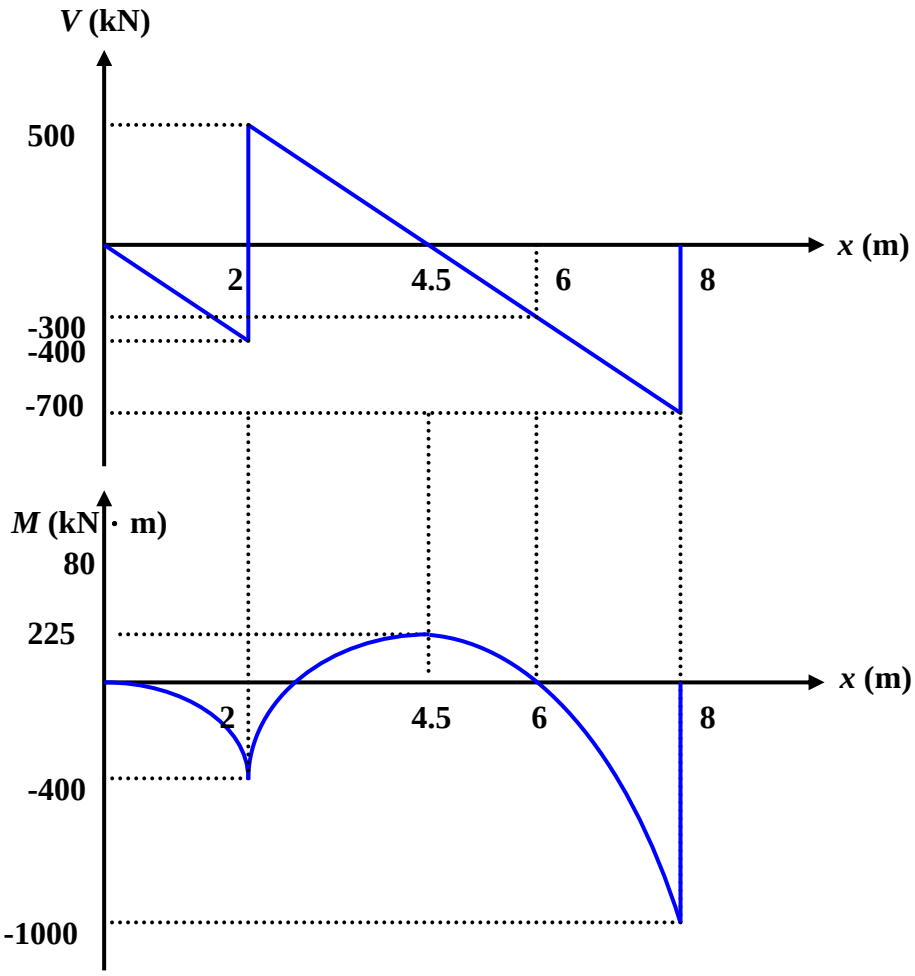


$$\sum M_C = 0 \quad \Rightarrow B_y \cdot 4 - (200 \cdot 6) \cdot 3 = 0$$
$$\Rightarrow B_y = 900 \ \text{(kN)}$$

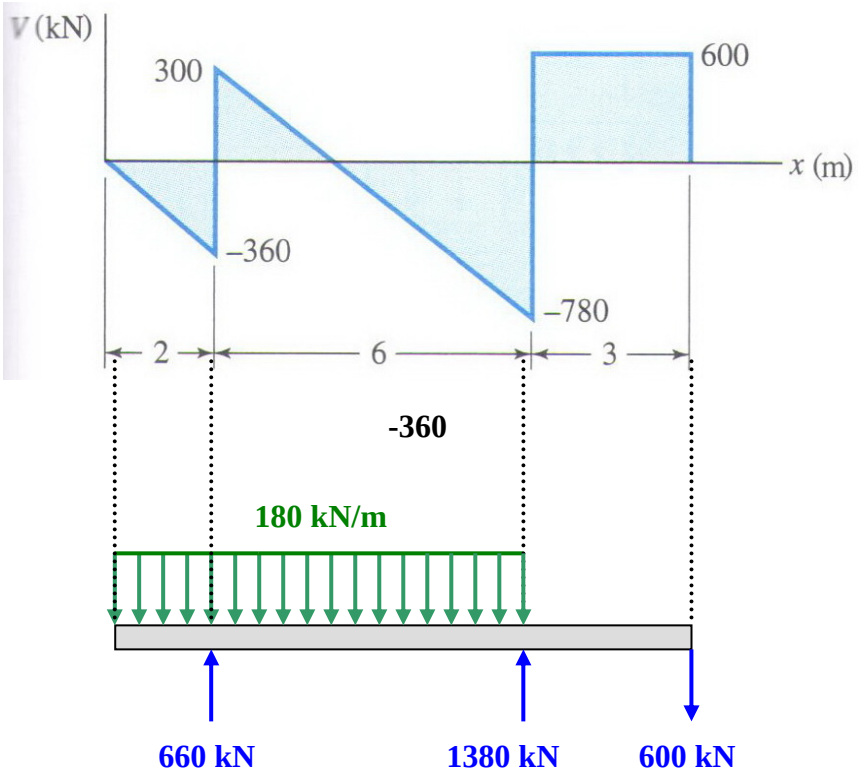
$$\sum F_y = 0 \quad \Rightarrow C_y = -300 \ \text{(kN)}$$

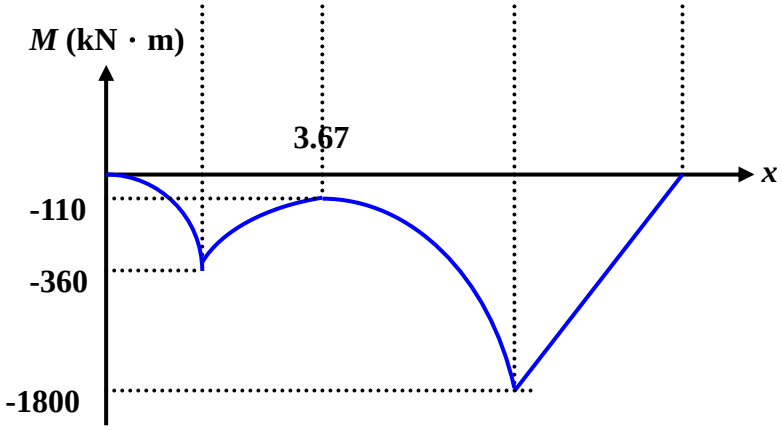


$$\sum F_y = 0 \quad \Rightarrow D_y = 700 \ \text{(kN)}$$
$$\sum M_D = 0 \quad \Rightarrow M - C_y \cdot 2 + (200 \cdot 2) \cdot 1 = 0$$
$$\Rightarrow M = -1000 \ \text{(kN} \cdot \text{m)}$$

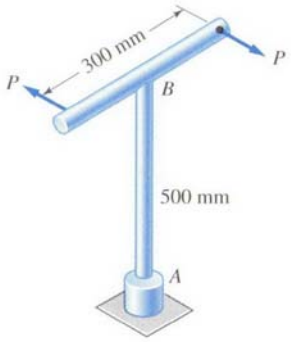


2. 依照所給的剪力圖，繪製相對應的負荷圖與彎矩圖。假設並無力偶作用於梁上。 (10%)





3. 大小為 *P* 的二力作用於下圖所示的鈑手上。已知鋼軸 *AB* 的直徑是15 mm。如軸內之剪應力不可超過120 MPa 且扭轉角不可超過5°。試求最大容許 *P* 值。(鋼的 $G = 80$ GPa) (15%)



$T = 0.3P$

$$J = \frac{\pi r^4}{2} = \frac{\pi(\frac{d}{2})^4}{2} = \frac{\pi d^4}{32}$$

$$\tau = \frac{Tc}{J} = \frac{T\cdot(\frac{d}{2})}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3} \quad \Rightarrow 120\cdot 10^6 = \frac{16\cdot(0.3P)}{\pi\cdot(0.015)^3} \quad \Rightarrow P = 265.07 \text{ (N)}$$

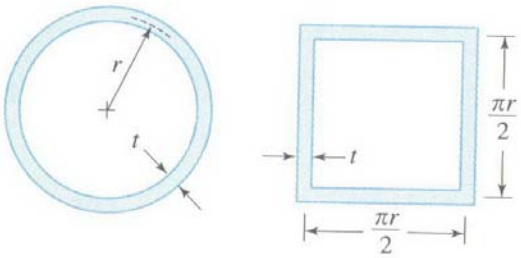
$$\phi = \frac{TL}{JG} \quad \Rightarrow 5\cdot\frac{\pi}{180} = \frac{(0.3P)\cdot 0.5}{\frac{\pi(0.015)^4}{32}\cdot 80\cdot 10^9} \quad \Rightarrow P = 231.32 \text{ (N)}$$

∴ *P* 為兩者取較小者，即 $P = 231.32$ (N)

4. 兩相同的金屬平版分別被製成如下圖所示之圓形及方形橫截面管件。若兩支管件都受到相同的扭矩，試求:

(1) 剪應力 $\tau_{圓形}/\tau_{方形}$ 之比值。 (10%)

(2) 扭轉角 $\phi_{圓形}/\phi_{方形}$ 之比值。 (10%)



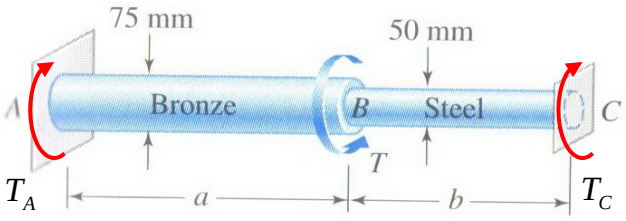
(1) $\tau = \dfrac{T}{2A_m} \quad \Rightarrow \dfrac{\tau_{圓形}}{\tau_{方形}} = \dfrac{(A_m)_{square}}{(A_m)_{circle}} = \dfrac{(\frac{\pi r}{2})^2}{\pi r^2} = \dfrac{\pi}{4}$

(2) $\phi = \dfrac{TL}{4A_m^2 G}\oint\dfrac{ds}{t} \quad \Rightarrow \dfrac{\tau_{圓形}}{\tau_{方形}} = \dfrac{(A_m^2)_{square}\cdot(\oint ds)_{circle}}{(A_m^2)_{circle}\cdot(\oint ds)_{square}} = \dfrac{(\frac{\pi r}{2})^4\cdot 2\pi r}{(\pi r^2)^2\cdot\frac{\pi r}{2}\cdot 4} = \dfrac{\pi^2}{16}$

5. 如下圖所示，複合軸兩端固定於剛性壁。其中青銅段 ***AB*** 直徑為 75 mm 及 $G = 35$ GPa 。而鋼段 ***BC*** 的直徑為 50 mm 及 $G = 83$ GPa 。

(1) 已知 $a = 2$ m 和 $b = 1.5$ m 。若青銅(Bronze)與鋼(Steel)之最大剪應力分別不超過 60 MPa 和 80 MPa ，試求所能施加之最大扭矩 ***T*** 。 **(10%)**

(2) 若使各材料的應力達其容許極限，試問扭矩 ***T*** 和 b/a 應為多少？ **(10%)**



(1) 由扭矩平衡可得: $T_A + T_C = T$

由相容方程 $\phi_{B/A} = -\phi_{C/B}$ $\Rightarrow \dfrac{T_A a}{J_{AB} G_{br}} = -\dfrac{(-T_C) b}{J_{BC} G_{st}}$

$$\Rightarrow T_A = \frac{b}{a}\frac{J_{AB} G_{br}}{J_{BC} G_{st}} T_C$$

$$\therefore \frac{b}{a}\frac{J_{AB} G_{br}}{J_{BC} G_{st}} T_C + T_C = T \quad \Rightarrow \frac{1.5}{2}\frac{75^4 \cdot 35}{50^4 \cdot 83} T_C + T_C = T$$

$$\Rightarrow T_C = \frac{T}{2.6011} = 0.3845T$$

$$\Rightarrow T_A = 0.6155T$$

$$\tau_{br} = \frac{T_A \cdot c_{AB}}{J_{AB}} \quad \Rightarrow 60 \cdot 10^6 = \frac{0.6155T \cdot (\frac{75}{2} \cdot 10^{-3})}{\frac{\pi}{2} \cdot (\frac{75}{2} \cdot 10^{-3})^4}$$

$$\Rightarrow T = 8.0749 \cdot 10^3 \ (\text{N} \cdot \text{m}) = 8.0749 \cdot 10^3 \ (\text{kN} \cdot \text{m})$$

$$\tau_{st} = \frac{T_C \cdot c_{BC}}{J_{BC}} \quad \Rightarrow 80 \cdot 10^6 = \frac{0.3845T \cdot (\frac{50}{2} \cdot 10^{-3})}{\frac{\pi}{2} \cdot (\frac{50}{2} \cdot 10^{-3})^4}$$

$$\Rightarrow T = 5.1066 \cdot 10^3 \ (\text{N} \cdot \text{m}) = 5.1066 \cdot 10^3 \ (\text{kN} \cdot \text{m})$$

$\therefore$ 能施加之最大扭矩 $T = 5.1066 \cdot 10^3 \ (\text{kN} \cdot \text{m})$

(2) 由 $\tau_{\max} = \dfrac{T \cdot c}{J}$ 與 $\phi = \dfrac{TL}{JG}$ 可得 $\phi = \dfrac{\tau_{\max} \cdot L}{c \cdot G}$

由相容方程 $\phi_{B/A} = -\phi_{C/B}$ $\Rightarrow \dfrac{(\tau_{\max})_{br} a}{c_{AB} G_{br}} = \dfrac{(\tau_{\max})_{st} b}{c_{BC} G_{st}}$

$$\Rightarrow \frac{b}{a} = \frac{(\tau_{\max})_{br}}{(\tau_{\max})_{st}} \cdot \frac{c_{BC}}{c_{AB}} \cdot \frac{G_{st}}{G_{br}} = \frac{60}{80} \cdot \frac{50}{75} \cdot \frac{83}{35}$$

$$\Rightarrow \frac{b}{a} = 1.1857$$

$$T = T_A + T_C = \frac{(\tau_{\max})_{AB} \cdot J_{AB}}{c_{AB}} + \frac{(\tau_{\max})_{BC} \cdot J_{BC}}{c_{BC}}$$

$$= \frac{60 \cdot 10^6 \cdot \frac{\pi}{2}(\frac{75}{2} \cdot 10^{-3})^4}{\frac{75}{2} \cdot 10^{-3}} + \frac{80 \cdot 10^6 \cdot \frac{\pi}{2}(\frac{50}{2} \cdot 10^{-3})^4}{\frac{50}{2} \cdot 10^{-3}}$$

$$= 6.9336 \cdot 10^3 \ (\text{N} \cdot \text{m})$$

$$= 6.9336 \ (\text{kN} \cdot \text{m})$$

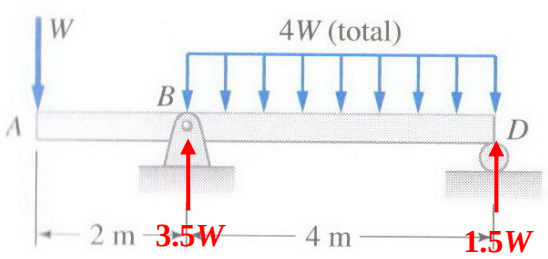
6. 樑承受集中負荷 W 和總合力 $4W$ 之均佈負荷如下左圖，其斷面如下右圖所示。

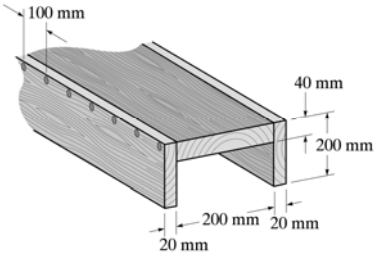
(1) 此樑斷面的中性軸位置？**(4%)**

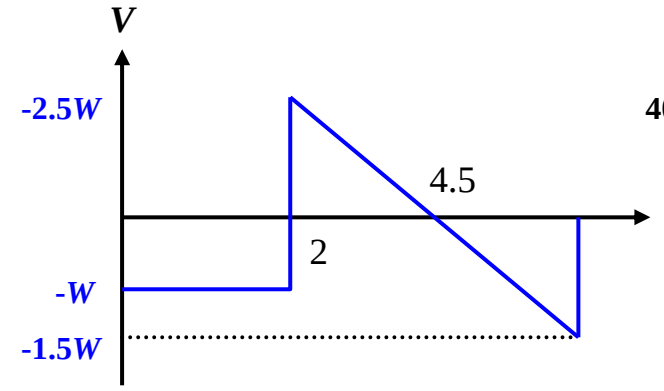
(2) 此斷面中性軸的面積慣性矩 $I = ?$ **(4%)**

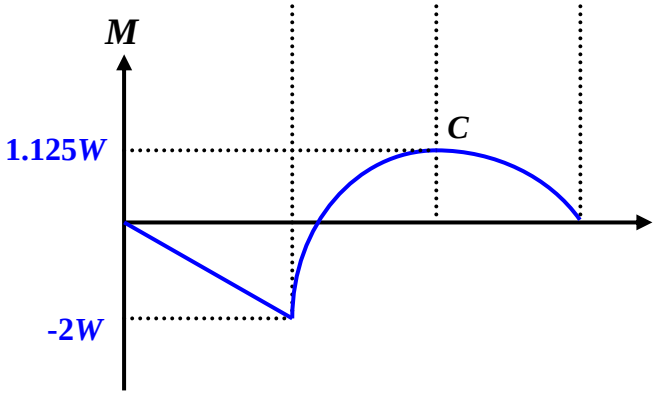
(3) 此樑的最大拉應力和最大壓應力分別為何？**(8%)**

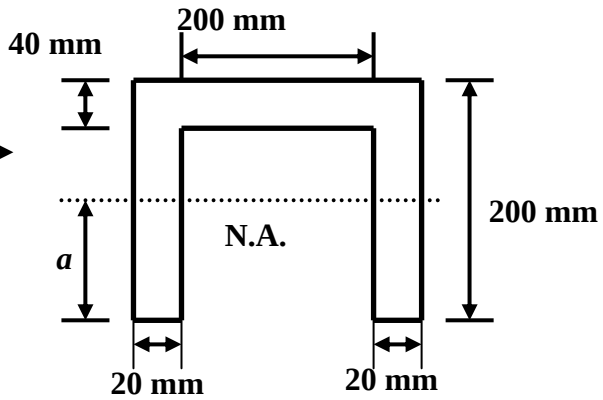
(4) 若容許拉伸應力為 60 MPa ，容許壓縮應力為 100 MPa ，試求 W 之最大容許值。 **(4%)**











(1) $\bar{y} = \dfrac{\sum y_i A_i}{\sum A_i} = \dfrac{0.18 \cdot (0.2 \cdot 0.04) + 0.1 \cdot (0.02 \cdot 0.2) \cdot 2}{(0.2 \cdot 0.04) + (0.02 \cdot 0.2) \cdot 2} = 0.14 \text{ (m)}$

$\therefore \quad a = \bar{y} = 0.14 \text{ (m)}$

(2) $I = \left[\dfrac{1}{12} \cdot 0.02 \cdot 0.2^3 + 0.02 \cdot 0.2 \cdot (0.14 - 0.1)^2 \right] \cdot 2$

$+ \dfrac{1}{12} \cdot 0.2 \cdot 0.04^3 + 0.2 \cdot 0.04 \cdot (0.18 - 0.14)^2$

$\Rightarrow I = \dfrac{160}{3} \cdot 10^{-6} = 53.3333 \cdot 10^{-6} \quad (\text{m}^4)$

(3) $\sigma = -\dfrac{My}{I}$

$(\sigma_B)_{top} = -\dfrac{(-2W) \cdot 0.06}{\dfrac{160}{3} \cdot 10^{-6}} = \dfrac{9W}{4} \cdot 10^3 \quad (\text{Pa}) = 2.25W \text{ (kPa)}$

$(\sigma_B)_{bottom} = -\dfrac{(-2W) \cdot (-0.14)}{\dfrac{160}{3} \cdot 10^{-6}} = -\dfrac{21W}{4} \cdot 10^3 \quad (\text{Pa}) = -5.25W \text{ (kPa)}$

$(\sigma_C)_{top} = -\dfrac{1.125W \cdot 0.06}{\dfrac{160}{3} \cdot 10^{-6}} = -\dfrac{81W}{64} \cdot 10^3 \quad (\text{Pa}) = -1.2656W \quad (\text{kPa})$

$(\sigma_C)_{bottom} = -\dfrac{1.125 \cdot (-0.14)}{\dfrac{160}{3} \cdot 10^{-6}} = \dfrac{189W}{64} \cdot 10^3 \quad (\text{Pa}) = 2.9531W \quad (\text{kPa})$

此樑的最大壓應力在 B 點底部為 $5.25W$ kPa

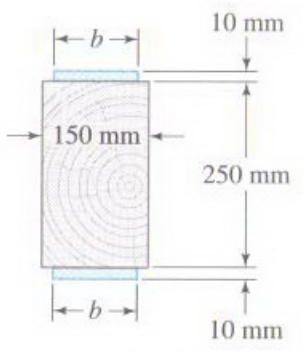
最大拉應力在 C 點底部為 $2.9531W$ MPa

(4) $-100 \cdot 10^6 = -\dfrac{21W}{4} \cdot 10^3 \quad \Rightarrow W = 19.0476 \cdot 10^3 \quad (\text{N}) = 19.0476 \quad (\text{kN})$

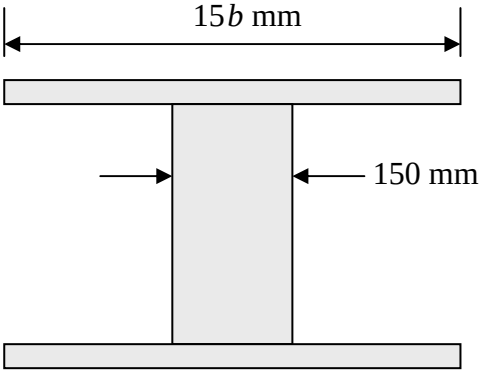
$60 \cdot 10^6 = \dfrac{189W}{64} \cdot 10^3 \quad \Rightarrow W = 20.3175 \cdot 10^3 \quad (\text{N}) = 20.3175 \quad (\text{kN})$

$\therefore W_{\max} = 19.0476 \quad (\text{kN})$

7. 木樑(wood)在底和頂端以寬度 *b* 的鋼板(steel)強化。若木材的容許應力為 10 MPa，鋼為120 MPa 且彈性模數比為 $E_{st}/E_{wd}=15$，試求能承受 40 kN·m 彎矩的最小寬度 *b*。 (15%)



將鋼轉換成木頭，可知轉換因子 $n=\dfrac{E_{st}}{E_{wd}}=15$



$$I=[\frac{1}{12}15b\cdot 10^3+(15b\cdot 10)\cdot 130^2]\cdot 2+\frac{1}{12}150\cdot 250^3$$
$$=(5.0725b+195.3125)\cdot 10^6 \ \ (\text{mm}^4)$$
$$=(5.0725b+195.3125)\cdot 10^{-6} \ \ (\text{m}^4)$$

$$\sigma_{wd}=\frac{Mc_{wd}}{I} \quad \Rightarrow 10\cdot 10^6=\frac{40\cdot 10^3\cdot(125\cdot 10^{-3})}{(5.0725b+195.3125)\cdot 10^{-6}} \quad \Rightarrow b=60.07\ (\text{mm})$$

$$\sigma_{st}=n\sigma'_{st}=\frac{nMc_{st}}{I} \quad \Rightarrow 120\cdot 10^6=\frac{15\cdot 40\cdot 10^3\cdot(135\cdot 10^{-3})}{(5.0725b+195.3125)\cdot 10^{-6}}$$

$\Rightarrow b=94.57\ (\text{mm})$

∴最小寬度 *b* 為94.57 mm