國立臺灣海洋大學河海工程學系 2020 材料力學 2AB 班第一次大考參考解答

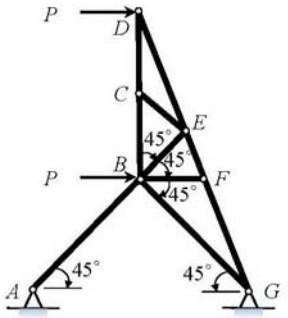
日期：2020 年 04 月 23 日　　姓名：＿＿＿＿＿＿　　學號：＿＿＿＿＿＿

1. 下圖之桁架由A−36合金鋼材組成，其彈性係數（modulus of elasticity）與降伏應力（yielding stress）分別為 $E = 200\ \text{GPa}$，$\sigma_Y = 250\ \text{MPa}$。所有桿件之斷面積$A$均為 $2.5\times10^3\ \text{mm}^2$，$AB$和$BG$長各為 4 m，$BC$和$CD$長各為 2 m。若受兩外力$P$作用（如下圖示），在所有桿件均無永久變形（permanent deformation）條件下，

   (1) 請指出所有零力構件。 (6%)

   (2) 試求出構件 AB、BC、BG 與 EF 之受力情況 (請註明受拉力或壓力)。 (8%)

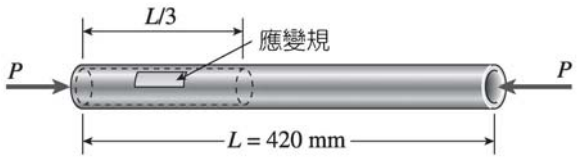
   (3) 試求最大作用力 P 之值。 (6%)



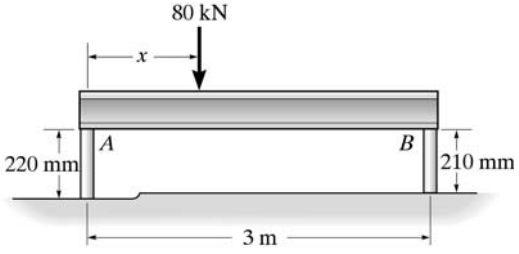
2. 長為 $L = 420\ \text{mm}$ 的圓形鋁管，受到壓力 P 作用(如下圖所示)，$L/3$ 中空段的外、內徑分別為 60 mm 和 35 mm。$2L/3$ 實心段直徑為 60 mm。一應變計裝在管的外表，以量取軸向的應變。

   (1) 若量得中空段的應變為 $\varepsilon_h = 470\times10^{-6}$，則實心段的應變 ε_s 為多少？ (10%)

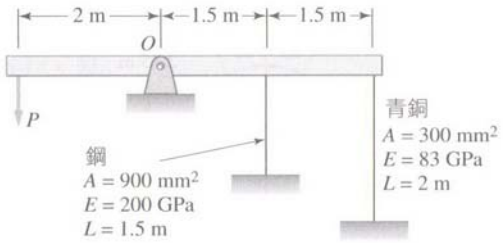
   (2) 求全桿的壓縮量 δ。 (10%)



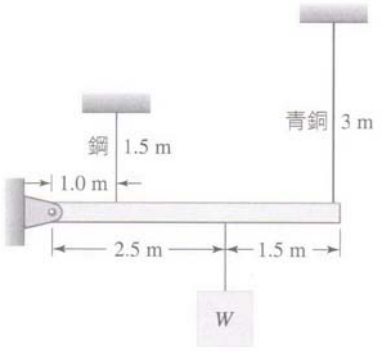
3. 剛性樑以水平靜止置於兩根 2014-T6 的鋁圓柱上，圓柱的未負載長度如圖示。若各圓柱的直徑為 30 mm，試求 80 KN 負載作用位置 x 使得樑維持水平。負載作用後 A 圓柱的新直徑為何？（$E_{al} = 73.1\ \text{GPa}$，$\nu_{al} = 0.35$）(20%)



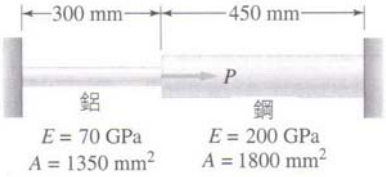
4. 重量可忽略的剛性桿件以銷固定在 O 點處，並與兩垂直桿連結如下圖所示。假設兩桿原先處無應力狀態。若鋼桿與青銅桿中之應力在分別不超過150 MPa 和 70 MPa 的要求下，試問可作用得最大負荷 P 為何？(20%)



5. 可忽略重量的剛性桿件受到如下圖所示的支撐。組合構件再初始時處於無應力狀態。若在負荷 $W=120\,\text{kN}$ 作用後，溫度上升 20°C，試求每根桿件之應力。（鋼：$E=200\,\text{GPa}$、$A=300\,\text{mm}^2$、$\alpha=11.7\times10^{-6}/°\text{C}$；青銅：$E=83\,\text{GPa}$、$A=1400\,\text{mm}^2$、$\alpha=18.9\times10^{-6}/°\text{C}$）。(20%)



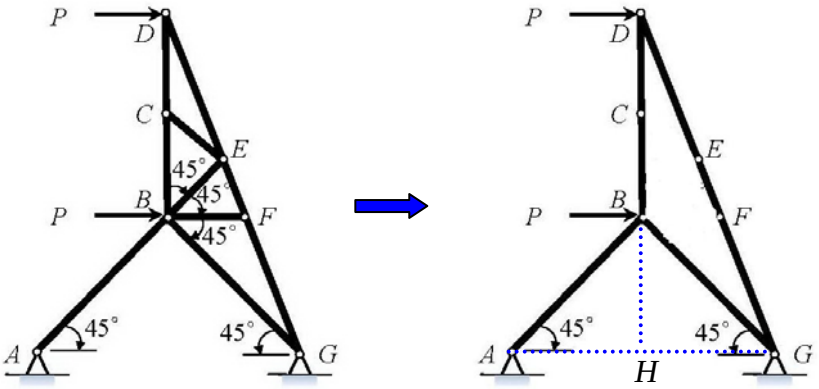
6. 如下圖所示複合桿牢固地連接於兩牆間。鋼材與鋁合金的降伏強度分別為 294 MPa 與 336 MPa，若施加負荷 $P=982.8\,\text{kN}$，然後移除，試求鋼材與鋁合金內之殘餘應力。 (20%)



參考解答:

1. (1) 零力桿件: BF、BE、CE

(2)



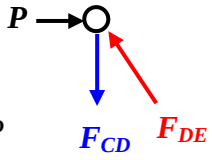
AB 長為 4 m ，故可知 $AH = HG = BH = 2\sqrt{2}$ (m)

DG 長為 $\sqrt{(4+2\sqrt{2})^2 + (2\sqrt{2})^2} = 7.3910$

由節點 D 可知

$$\sum F_x = 0 \quad \Rightarrow F_{DE} \cdot \frac{2\sqrt{2}}{7.3910} - P = 0 \quad \Rightarrow F_{DE} = 2.6131P$$

$$\sum F_y = 0 \quad \Rightarrow F_{DE} \cdot \frac{4+2\sqrt{2}}{7.3910} - F_{CD} = 0 \quad \Rightarrow F_{CD} = 2.4142P$$



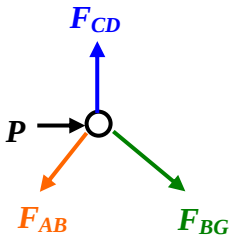
故可得 $F_{BC} = F_{CD} = 2.4142P$

$F_{EF} = F_{FG} = F_{DE} = 2.6131P$

由節點 B 可知

$$\sum F_x = 0 \quad \Rightarrow F_{BG} \cdot \frac{1}{\sqrt{2}} + P - F_{AB} \cdot \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow F_{AB} - F_{BG} = \sqrt{2}P \quad \ldots\ldots \text{(a)}$$



$$\sum F_y = 0 \quad \Rightarrow F_{BG} \cdot \frac{1}{\sqrt{2}} + F_{AB} \cdot \frac{1}{\sqrt{2}} - F_{CD} = 0$$

$$\Rightarrow F_{AB} + F_{BG} = 3.4142P \quad \ldots\ldots \text{(b)}$$

由(a)+(b)可得 $F_{AB} = 2.4142P$

由(b)-(a)可得 $F_{BG} = P$

(3) 由於 A-36 鋼 $\sigma_Y = 200$ MPa 且桿之斷面積為 2.5×10^3 mm^2 $= 2.5\times10^{-3}$ m^2

可知 $\sigma_Y = \dfrac{F_{DE}}{A} \quad \Rightarrow 250\cdot10^6 = \dfrac{2.6131P}{2.5\cdot10^{-3}}$

$$\Rightarrow P = 239.1795\cdot10^3 \text{ (N)} = 239.1795 \text{ (kN)}$$

2. (1) 中空段的應變為 $\varepsilon_h = 470\times10^{-6}$ ， $A_h = \frac{\pi}{4}(60^2 - 35^2)$ mm^2 ，

假設鋁管楊氏模數為 E，

可得中空段應力為 $\sigma_h = E\cdot\varepsilon_h$ ，受力 $P = \sigma_h \cdot A_h$

∴ 實心段受力亦為 $P = \sigma_h \cdot A_h = E\cdot\varepsilon_h \cdot A_h$

應力為 $\sigma_s = \frac{P}{A_s} = \frac{E\cdot\varepsilon_h\cdot A_h}{A_s}$

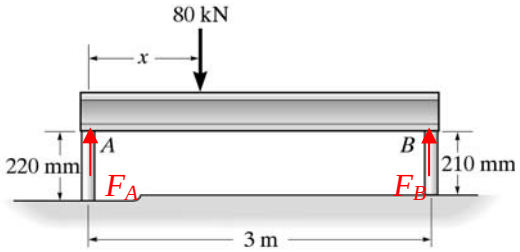
應變為 $\varepsilon_s = \frac{\sigma_s}{E} = \frac{\varepsilon_h\cdot A_h}{A_s} = \frac{470\cdot10^{-6}\cdot\frac{\pi}{4}(60^2-35^2)}{\frac{\pi}{4}(60^2)} = 310.07\cdot10^{-6}$

(2) 全桿的壓縮量 $\delta = \sigma_h\cdot\frac{L}{3} + \sigma_s\cdot\frac{2L}{3}$

$= 470\cdot10^{-6}\cdot140 + 310.07\cdot10^{-6}\cdot280$

$= 0.1526$ (mm)

3.



$\sum M_A = 0 \quad \Rightarrow 80\cdot x - F_B\cdot3 = 0 \quad \Rightarrow F_B = \frac{80}{3}x$

$\sum M_B = 0 \quad \Rightarrow 80\cdot(3-x) - F_A\cdot3 = 0 \quad \Rightarrow F_A = 80\cdot(1-\frac{x}{3})$

又80 KN 負載作用後，樑要維持水平，表示兩鋁圓柱的壓縮量要相同

即 $\delta_A = \delta_B \Rightarrow \frac{F_A\cdot220}{AE} = \frac{F_B\cdot210}{AE} \Rightarrow F_A = \frac{21}{22}F_B$

故可得 $80\cdot(1-\frac{x}{3}) = \frac{21}{22}\cdot\frac{80}{3}x \Rightarrow x = 1.5349$ (m) $\Rightarrow F_A = 39.0693$ (kN)

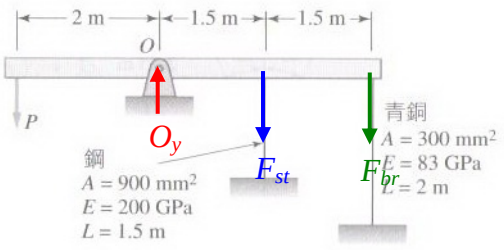
∴ $\sigma_A = \frac{F_A}{A} = \frac{39.0693\cdot10^3}{\frac{\pi}{4}\cdot30^2\cdot10^{-6}} = 55.2718\cdot10^6$ (Pa) $= 55.2718$ (MPa)

$\varepsilon_{long} = -\frac{\sigma_A}{E_{al}} = -\frac{55.2718\cdot10^6}{73.1\cdot10^9} = -0.7561\cdot10^{-3}$

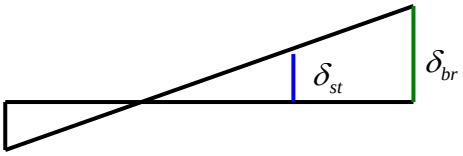
$\nu_{al} = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} \Rightarrow \varepsilon_{lat} = -\nu\cdot\varepsilon_{long} = 0.35\cdot0.7561\cdot10^{-3} = 0.2646\cdot10^{-3}$

$d'_A = d_A(1+\varepsilon_{lat}) = 30(1+0.2646\cdot10^{-3}) = 30.0079$ (mm)

4.



$$\sum M_o = 0 \quad \Rightarrow 1.5 \cdot F_{st} + 3 \cdot F_{br} - 2P = 0 \quad \Rightarrow 1.5F_{st} + 3F_{br} = 2P \ldots \text{(a)}$$



由幾何變形可知相容方程為 $2\delta_{st} = \delta_{br}$

$$\Rightarrow 2\frac{F_{st} \cdot L_{st}}{E_{st} \cdot A_{st}} = \frac{F_{br} \cdot L_{br}}{E_{br} \cdot A_{br}}$$

$$\Rightarrow 2\frac{F_{st} \cdot 1.5}{200 \cdot 10^9 \cdot 900 \cdot 10^{-6}} = \frac{F_{br} \cdot 2}{83 \cdot 10^9 \cdot 300 \cdot 10^{-6}}$$

$$\Rightarrow F_{br} = \frac{83}{400} F_{st} \quad \ldots \text{(b)}$$

代回(a)可得 $F_{st} = \frac{800}{849} P = 0.9423P$

代回(b)可得 $F_{br} = \frac{166}{849} P = 0.1955P$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} \quad \Rightarrow F_{st} = \sigma_{st} A_{st} \quad \Rightarrow \frac{800}{849} P = 150 \cdot 10^6 \cdot 900 \cdot 10^{-6}$$

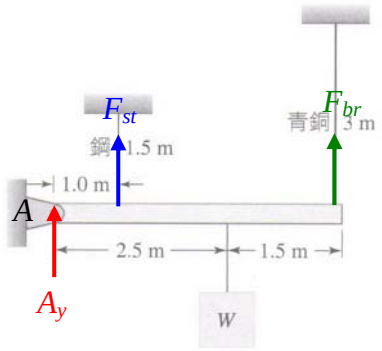
$$\Rightarrow P = 143268.75 \ \text{(N)} \cong 143.27 \ \text{(kN)}$$

$$\sigma_{br} = \frac{F_{br}}{A_{br}} \quad \Rightarrow F_{br} = \sigma_{br} A_{br} \quad \Rightarrow \frac{166}{849} P = 70 \cdot 10^6 \cdot 300 \cdot 10^{-6}$$

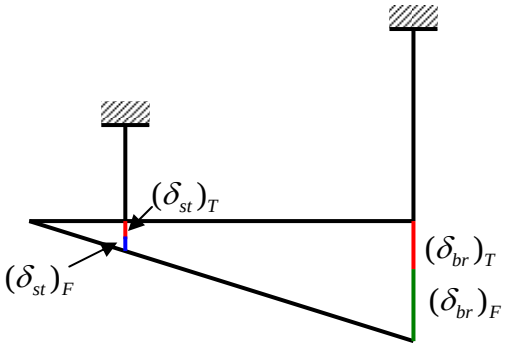
$$\Rightarrow P = 107403.6145 \ \text{(N)} \cong 107.40 \ \text{(kN)}$$

∴ 最大負荷 P 為 107.40 kN

5.



$$\sum M_A = 0 \quad \Rightarrow 1\cdot F_{st} + 4\cdot F_{br} - 2.5W = 0 \quad \Rightarrow F_{st} + 4F_{br} = 300\cdot 10^3 \ldots\text{(a)}$$



由幾何變形可知相容方程為 $4[(\delta_{st})_T + (\delta_{st})_F] = (\delta_{br})_T + (\delta_{br})_F$

$$\therefore \quad 4\left(\alpha_{st}\cdot\Delta T\cdot L_{st} + \frac{F_{st}\cdot L_{st}}{A_{st}\cdot E_{st}}\right) = \alpha_{br}\cdot\Delta T\cdot L_{br} + \frac{F_{br}\cdot L_{br}}{A_{br}\cdot E_{br}}$$

$$\Rightarrow 4\left(11.7\cdot 10^{-6}\cdot 20\cdot 1.5 + \frac{F_{st}\cdot 1.5}{300\cdot 10^{-6}\cdot 200\cdot 10^{9}}\right)$$

$$= 18.9\cdot 10^{-6}\cdot 20\cdot 3 + \frac{F_{br}\cdot 3}{1400\cdot 10^{-6}\cdot 83\cdot 10^{9}}$$

$$\Rightarrow \frac{F_{br}}{1162} - \frac{F_{st}}{300} = 9 \ldots\text{(b)}$$

將(a)代入(b)可得 $\left(\frac{1}{1162} + \frac{1}{75}\right)F_{br} = 1009$

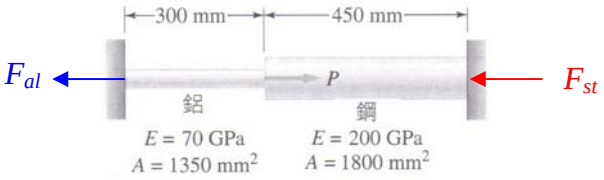
$$\Rightarrow F_{br} = 71086.7852 \text{ (N)} \cong 71.09 \text{ (kN)}$$

代回(a)可得 $F_{st} = 15652.8698 \text{ (N)} \cong 15.65 \text{ (kN)}$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{15652.8698}{300\cdot 10^{-6}} = 52.1762\cdot 10^{6} \text{ (Pa)} \cong 52.1762 \text{ (MPa)}$$

$$\sigma_{br} = \frac{F_{br}}{A_{br}} = \frac{71086.7852}{1400\cdot 10^{-6}} = 50.7763\cdot 10^{6} \text{ (Pa)} \cong 50.7763 \text{ (MPa)}$$

6.



$$\sum F_x = 0 \quad \Rightarrow F_{al} + F_{st} = P \quad \ldots(a)$$

相容方程：$\delta_{al} - \delta_{st} = 0$

$$\Rightarrow \frac{F_{al} \cdot L_{al}}{A_{al} \cdot E_{al}} - \frac{F_{st} \cdot L_{st}}{A_{st} \cdot E_{st}} = 0$$

$$\Rightarrow \frac{F_{al} \cdot 300}{1350 \cdot 70} - \frac{F_{st} \cdot 450}{1800 \cdot 200} = 0$$

$$\Rightarrow F_{al} = \frac{63}{160} F_{st} \quad \ldots(b)$$

將(a)代入(b)可得 $F_{st} = \frac{160}{223} P = 705.1480$ (kN)

代回(b)可得 $F_{al} = \frac{63}{223} P = 277.6520$ (kN)

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{705.1480 \cdot 10^3}{1800 \cdot 10^{-6}} = 391.7489 \cdot 10^6 \text{ (Pa)} \cong 391.75 \text{ (MPa)}$$

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{277.6520 \cdot 10^3}{1350 \cdot 10^{-6}} = 205.6681 \cdot 10^6 \text{ (Pa)} \cong 205.67 \text{ (MPa)}$$

∵ $\sigma_{st} > 294$ MPa 、$\sigma_{al} > 336$ MPa

∴ 可知鋼材已進入塑性狀態，鋁合金仍在彈性狀態

故須進行應力重分配

$$F'_{st} = (\sigma_Y)_{st} \cdot A_{st} = 294 \cdot 10^6 \cdot 1800 \cdot 10^{-6} = 529200 \text{ (N)} = 529.2 \text{ (kN)}$$

$$F'_{al} = 982.8 - 529.2 = 453.6 \text{ (kN)}$$

$$\sigma'_{st} = 294 \text{ (MPa)}$$

$$\sigma'_{al} = \frac{F'_{al}}{A_{al}} = \frac{453.6 \cdot 10^3}{1350 \cdot 10^{-6}} = 336 \cdot 10^6 \text{ (Pa)} = 336 \text{ (MPa)}$$

$$(\sigma_{st})_r = 391.75 - 294 = 97.75 \text{ (MPa)} \quad \text{(拉力)}$$

$$(\sigma_{al})_r = 336 - 205.67 = 130.33 \text{ (MPa)} \quad \text{(拉力)}$$