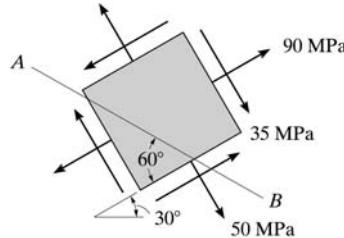


日期：2019 年 06 月 20 日 姓名：_____ 學號：_____

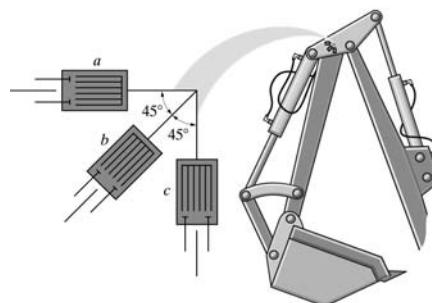
1. 將構件內一點的應力狀態標示於元素上，如下圖所示。

- (1) 試畫其莫爾圓。 (6%)
- (2) 此點的主應力、最大平面剪應力與其各別對應之方位為何？ (8%)
- (3) 作用在 AB 斜面的應力分量為何？ (6%)



2. -45° 應變菊花座係貼在挖土機連桿上。各量規上讀數為： $\varepsilon_a = 650 \times 10^{-6}$, $\varepsilon_b = -300 \times 10^{-6}$, $\varepsilon_c = 480 \times 10^{-6}$ 。試求：

- (1) 主應變。 (10%)
- (2) 最大同平面剪應變及相關平均正向應變。 (10%)

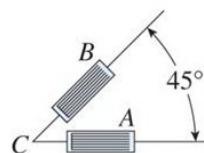
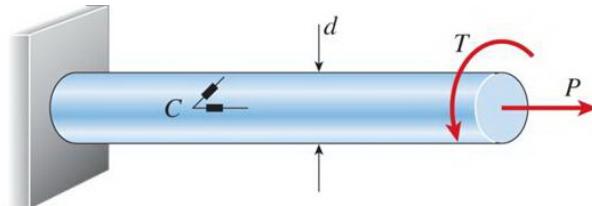


3. 一直徑 $d = 32\text{ mm}$ 的實心圓桿受到一軸向力 P 及扭矩 T 作用，如下圖所示。

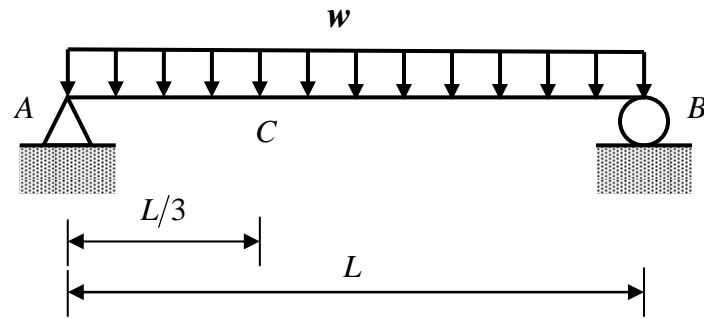
應變規 A 及 B 裝在桿件表面上，得出讀數 $\varepsilon_A = 140 \times 10^{-6}$ 及 $\varepsilon_B = -60 \times 10^{-6}$ 。

桿件由鋼材製成， $E = 210\text{ GPa}$ 及 $\nu = 0.29$ 。

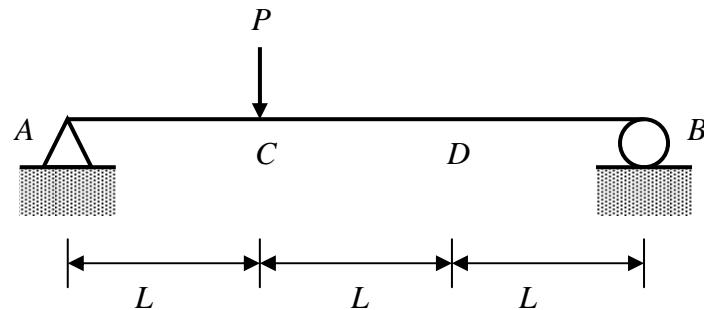
- (1) 求軸向力 P 及扭矩 T 。 (10%)
- (2) 求桿件內的最大剪應變 γ_{\max} 及最大剪應力 τ_{\max} 。 (10%)



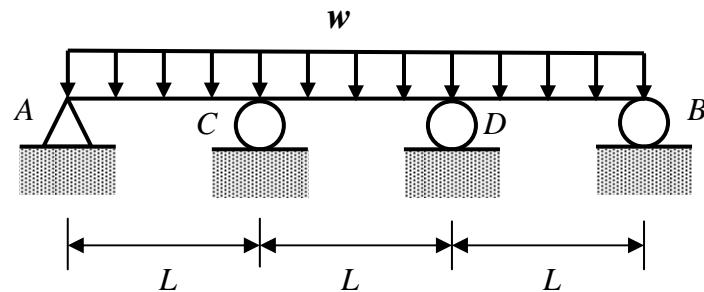
4. 試以積分法求下圖簡支樑的彈性曲線方程式並計算 θ_A 、 v_C 與 v_{\max} °。(20%)



5. 試以面積力矩法求下圖懸臂樑之 θ_A 、 θ_B 、 v_C 與 v_D °。(20%)



6. 試求下圖 A 與 C 支承反力大小並求 θ_A °。(10%)



7. (1) 上完了一學期的材料力學，對於這門課在學習上有何心得或感想？(5%)

(2) 對於老師的教學方式或是要如何協助同學們學習好這門課有何建議？(5%)

(參考公式)

平面應力轉換方程式

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\text{扭轉公式: } \tau = \frac{T\rho}{J}, \quad \text{彎曲公式: } \sigma = -\frac{My}{I}, \quad \text{剪力公式: } \tau = \frac{VQ}{It}$$

$$\text{應變-應力關係式: } \varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)], \quad \varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

平面應變轉換方程式

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$

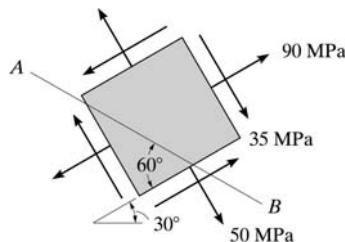
參考解答：

1. 將構件內一點的應力狀態標示於元素上，如下圖所示。

(1) 試畫其莫爾圓。 (6%)

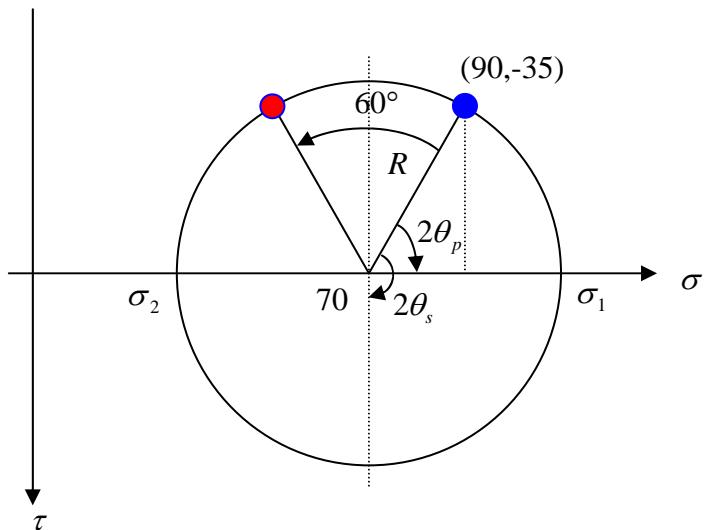
(2) 此點的主應力、最大平面剪應力與其各別對應之方位為何？ (8%)

(3) 作用在 AB 斜面的應力分量為何？ (6%)



$$(1) \sigma_x = 90 \text{ MPa}, \sigma_y = 50 \text{ MPa}, \tau_{xy} = -35 \text{ MPa}$$

$$\Rightarrow \sigma_{avg} = \frac{90 + 50}{2} = 70 \text{ (MPa)}$$



$$(2) R = \sqrt{(90 - 70)^2 + (-35)^2} = 40.31$$

$$\text{主應力: } \sigma_1 = 70 + 40.31 = 110.31 \text{ (MPa)}$$

$$\sigma_2 = 70 - 40.31 = 29.69 \text{ (MPa)}$$

$$\tan 2\theta_p = \frac{35}{20} \Rightarrow 2\theta_p = \tan^{-1} \frac{35}{20} = 1.0517 \text{ (rad)} = 60.26^\circ$$

$$\Rightarrow \theta_p = 30.13^\circ \text{ (順時鐘方向旋轉)}$$

$$\text{最大平面剪應力: } \tau_{max} = R = 40.31 \text{ (MPa)}$$

$$2\theta_s = 2\theta_p + 90^\circ \Rightarrow \theta_s = 75.13^\circ \text{ (順時鐘方向旋轉)}$$

(3) AB 斜面為元素逆時鐘方向旋轉 30°，故在莫爾圓上為逆時鐘方向旋轉 60°

$$(\sigma_x)_{AB} = 70 - 40.31 \cdot \sin(60^\circ - 29.74^\circ) = 49.69 \text{ (MPa)}$$

$$(\tau_{xy})_{AB} = -40.31 \cdot \cos(60^\circ - 29.74^\circ) = -34.82 \text{ (MPa)}$$

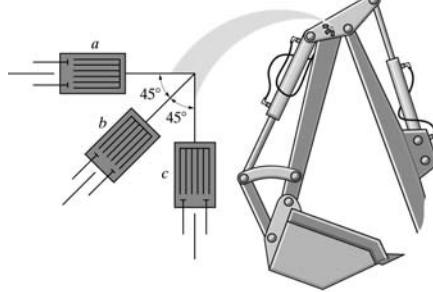
2. -45° 應變菊花座係貼在挖土機連桿上。各量規上讀數為： $\varepsilon_a = 650 \times 10^{-6}$,

$$\varepsilon_b = -300 \times 10^{-6}, \quad \varepsilon_c = 480 \times 10^{-6}$$

試求：

(1) 主應變。(10%)

(2) 最大同平面剪應變及相關平均正向應變。(10%)



$$\varepsilon_a = 650 \times 10^{-6} = \varepsilon_x \cdot \cos^2 180^\circ + \varepsilon_y \cdot \sin^2 180^\circ + \gamma_{xy} \sin 180^\circ \cdot \cos 180^\circ$$

$$\Rightarrow \varepsilon_x = 650 \times 10^{-6}$$

$$\varepsilon_b = -300 \times 10^{-6} = \varepsilon_x \cdot \cos^2 225^\circ + \varepsilon_y \cdot \sin^2 225^\circ + \gamma_{xy} \sin 225^\circ \cdot \cos 225^\circ$$

$$\Rightarrow \varepsilon_x + \varepsilon_y + \gamma_{xy} = -600 \times 10^{-6}$$

$$\varepsilon_c = 480 \times 10^{-6} = \varepsilon_x \cdot \cos^2 270^\circ + \varepsilon_y \cdot \sin^2 270^\circ + \gamma_{xy} \sin 270^\circ \cdot \cos 270^\circ$$

$$\Rightarrow \varepsilon_y = 480 \times 10^{-6} = 1732 \times 10^{-6}$$

$$\therefore \text{可得 } \gamma_{xy} = -1730 \times 10^{-6}$$

$$(1) \quad \varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 1437.17 \times 10^{-6} \text{ and } -304.17 \times 10^{-6}$$

$$(2) \quad \frac{\gamma_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 869.17 \times 10^{-6} \Rightarrow \gamma_{\max} = 1738.34 \times 10^{-6}$$

$$\varepsilon_{avg} = \frac{\varepsilon_1 + \varepsilon_2}{2} = 565 \times 10^{-6}$$

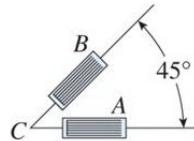
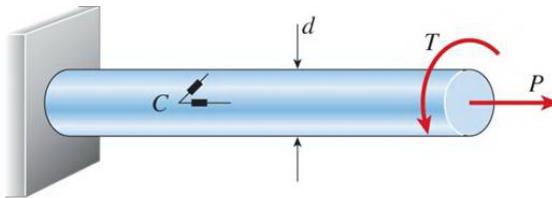
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桿件由鋼材製成， $E = 210\text{ GPa}$ 及 $\nu = 0.29$ 。

(1) 求軸向力 P 及扭矩 T 。 (10%)

(2) 求桿件內的最大剪應變 γ_{\max} 及最大剪應力 τ_{\max} 。 (10%)



$$(1) \text{ 由 } \sigma_x = \frac{P}{A} \text{ 並且 } \sigma_x = E \cdot \varepsilon_x \text{ 可知 } \frac{P}{A} = E \cdot \varepsilon_x$$

又由 A 應變規讀數 $\varepsilon_A = \varepsilon_x = 140 \cdot 10^{-6}$ 可得

$$\frac{P}{\pi \cdot \left(\frac{32}{2}\right)^2 \cdot 10^{-6}} = 210 \cdot 10^9 \cdot 140 \cdot 10^{-6} \Rightarrow P = 23644.88 (\text{N}) = 23.64 (\text{kN})$$

$$\text{由 } \varepsilon_y = \frac{1}{E}(\sigma_y - \nu \sigma_x) = -\frac{\nu}{E} \sigma_x = -\nu \cdot \varepsilon_x = -40.60 \cdot 10^{-6}$$

$$\begin{aligned} \text{由應變轉換公式可知 } \varepsilon_B &= \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &\Rightarrow -60 \cdot 10^{-6} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 90^\circ + \frac{\gamma_{xy}}{2} \sin 90^\circ \\ &\Rightarrow \gamma_{xy} = -219.40 \cdot 10^{-6} \end{aligned}$$

$$\text{又 } \tau = \frac{T\rho}{J} \text{ 且 } \tau = G\gamma_{xy} \text{ 可知 } \frac{T\rho}{J} = G\gamma_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy}$$

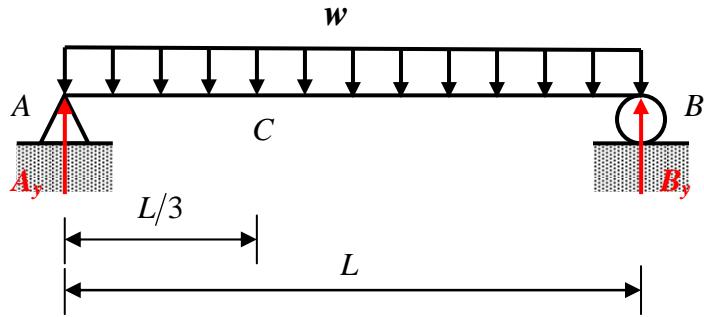
$$\Rightarrow T = \frac{\pi \left(\frac{32}{2}\right)^3 \cdot 10^{-9} \cdot 210 \cdot 10^9}{4(1+0.29)} \cdot (-219.40 \cdot 10^{-6}) = -114.90 (\text{N} \cdot \text{m})$$

(2) 由 $\varepsilon_x = 140 \cdot 10^{-6}$, $\varepsilon_y = -40.60 \cdot 10^{-6}$, $\Rightarrow \gamma_{xy} = -219.40 \cdot 10^{-6}$ 可得

$$\frac{\gamma_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 142.09 \cdot 10^{-6} \Rightarrow \gamma_{\max} = 284.18 \cdot 10^{-6}$$

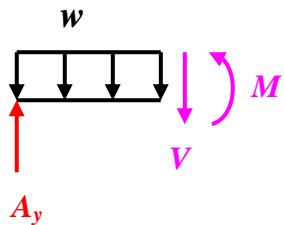
$$\tau_{\max} = G\gamma_{\max} = \frac{E}{2(1+\nu)} \gamma_{\max} = 23.13 \cdot 10^6 (\text{Pa}) = 23.13 (\text{MPa})$$

4. 試以積分法求下圖簡支樑的彈性曲線方程式並計算 θ_A 、 v_C 與 v_{\max} 。(20%)



\because 幾何對稱

$$\therefore \text{可知 } A_y = B_y = \frac{wL}{2}$$



$$\begin{aligned} \sum M_o &= 0 \Rightarrow M + wx \cdot \frac{x}{2} - A_y \cdot x = 0 \\ &\Rightarrow M = \frac{wL}{2}x - \frac{wx^2}{2} \end{aligned}$$

$$\begin{aligned} \frac{d^2v}{dx^2} &= \frac{M}{EI} \Rightarrow \frac{dv}{dx} = \theta = \frac{1}{EI} \left(\frac{wL}{4}x^2 - \frac{w}{6}x^3 + c_1 \right) \\ &\Rightarrow v = \frac{1}{EI} \left(\frac{wL}{12}x^3 - \frac{w}{24}x^4 + c_1x + c_2 \right) \end{aligned}$$

由邊界條件可得 $v(0) = 0 \Rightarrow c_2 = 0$

$$\text{由幾何對稱條件可得 } \theta\left(\frac{L}{2}\right) = 0 \Rightarrow c_1 = -\frac{wL^3}{24EI}$$

$$\therefore \text{彈性曲線方程: } v(x) = \frac{wx}{24EI} (2Lx^2 - x^3 - L^3)$$

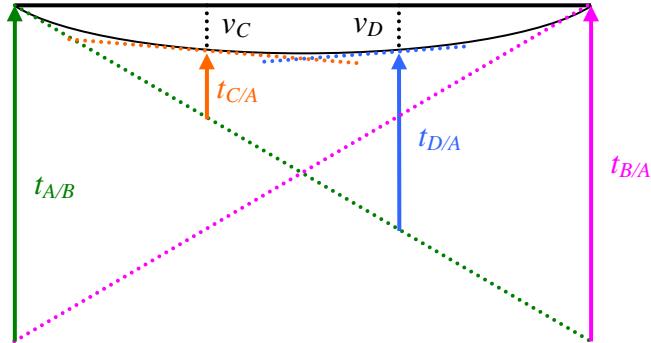
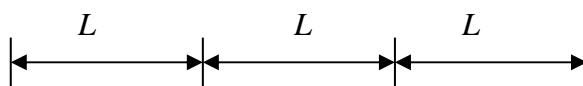
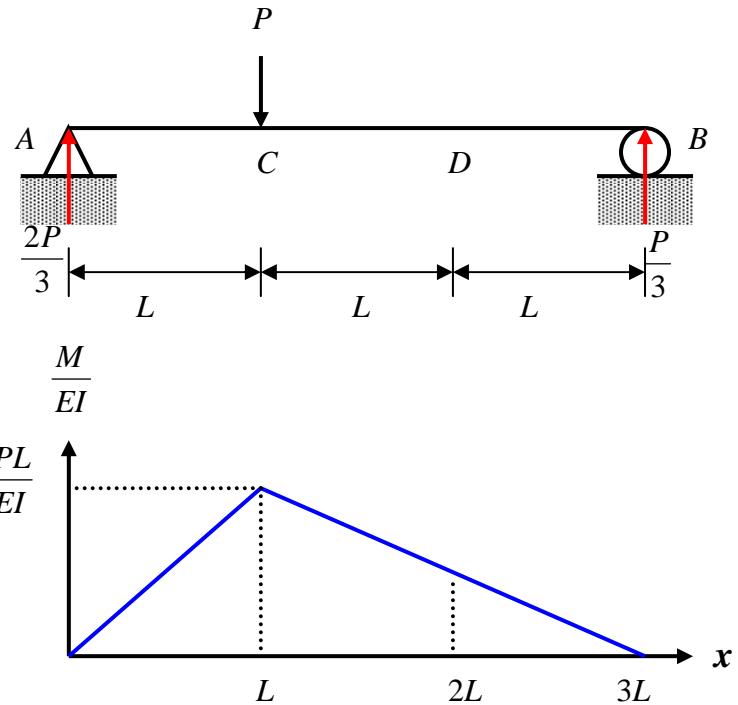
$$\Rightarrow \theta(x) = \frac{w}{24EI} (6Lx^2 - 4x^3 - L^3)$$

$$\theta_A = \theta(0) = -\frac{wL^3}{24EI} \quad (\text{負號表示順時鐘方向旋轉})$$

$$v_C = v\left(\frac{L}{3}\right) = -\frac{11wL^4}{972EI} \quad (\text{負號表示位移向下})$$

$$v_{\max} = v\left(\frac{L}{2}\right) = -\frac{5wL^4}{384EI} \quad (\text{負號表示位移向下})$$

5. 試以面積力矩法求下圖懸臂樑之 θ_A 、 θ_B 、 v_C 與 v_D 。 (20%)



$$\text{令 } \delta_C = v_C + t_{C/A}, \quad \delta_D = v_D + t_{D/A}$$

由面積-力矩法第二定理可知

$$t_{A/B} = \frac{1}{2} \cdot L \cdot \frac{2PL}{3EI} \cdot \frac{2L}{3} + \frac{1}{2} \cdot 2L \cdot \frac{2PL}{3EI} \cdot (L + \frac{2L}{3}) = \frac{12PL^3}{9EI} = \frac{4PL^3}{3EI}$$

$$t_{B/A} = \frac{1}{2} \cdot L \cdot \frac{2PL}{3EI} \cdot (2L + \frac{L}{3}) + \frac{1}{2} \cdot 2L \cdot \frac{2PL}{3EI} \cdot \frac{4L}{3} = \frac{15PL^3}{9EI} = \frac{5PL^3}{3EI}$$

$$t_{D/A} = \frac{1}{2} \cdot L \cdot \frac{2PL}{3EI} \cdot (L + \frac{L}{3}) + \frac{1}{2} \cdot L \cdot \frac{PL}{3EI} \cdot \frac{2L}{3} + L \cdot \frac{PL}{3EI} \cdot \frac{L}{2} = \frac{13PL^3}{18EI}$$

$$t_{C/A} = \frac{1}{2} \cdot L \cdot \frac{2PL}{3EI} \cdot \frac{L}{3} = \frac{PL^3}{9EI}$$

$$\theta_A = \frac{t_{B/A}}{3L} = \frac{5PL^2}{9EI} \text{ (順時鐘方向旋轉)}$$

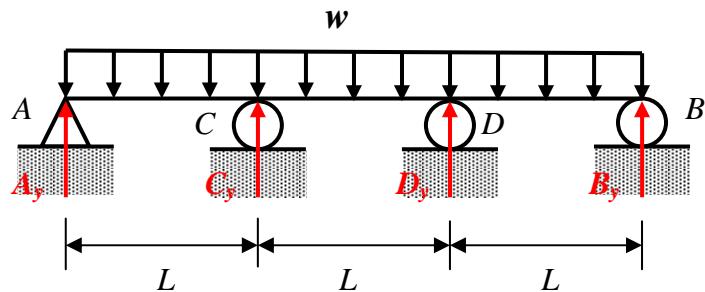
$$\theta_B = \frac{t_{A/B}}{3L} = \frac{4PL^2}{9EI} \text{ (順時鐘方向旋轉)}$$

由圖可看出 $\delta_C = \frac{1}{3}t_{B/A} = \frac{5PL^3}{9EI}$ 與 $\delta_D = \frac{2}{3}t_{B/A} = \frac{10PL^3}{9EI}$

$$\therefore v_C = \delta_C - t_{C/A} = \frac{4PL^3}{9EI} \text{ (向下)}$$

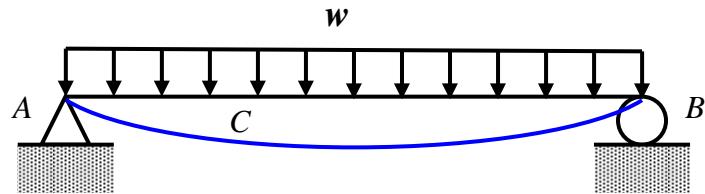
$$v_D = \delta_D - t_{D/A} = \frac{7PL^3}{18EI} \text{ (向下)}$$

6. 試求下圖 A 與 C 支承反力大小並求 θ_A 。 (10%)

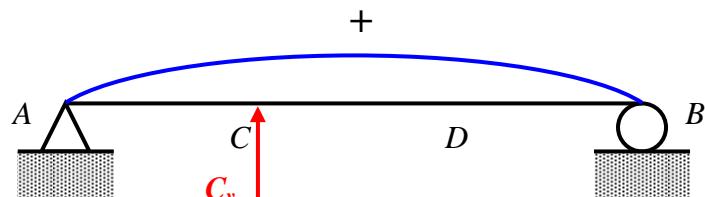


\because 幾何對稱

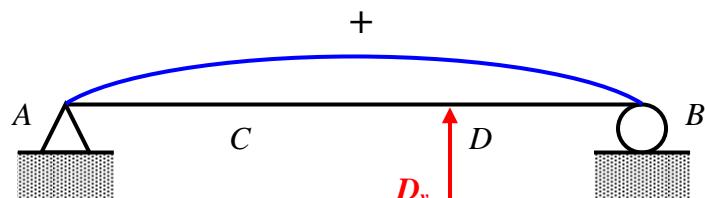
\therefore 可知 $A_y = B_y$, $C_y = D_y$ 並可將問題分解為



圖(a)



圖(b)



圖(c)

第 4 題彈性曲線方程: $v(x) = \frac{wx}{24EI}(2Lx^2 - x^3 - L^3)$

由於圖(a)之樑長為 $3L$ ，故將方程中 L 以 $3L$ 取代，

並代入 $x = L$ 可得圖(a) C 點位移 $\bar{v}_C = -\frac{22wL^4}{24EI}$, A 點轉角 $\bar{\theta}_A = -\frac{27wL^3}{24EI}$

由第 5 題可知圖(b) C 點位移 $v'_C = \frac{4C_y L^3}{9EI}$, A 點轉角為 $\theta'_A = \frac{5C_y L^2}{9EI}$

同理：由第 5 題可知圖(c) C 點位移 $v''_C = \frac{7D_y L^3}{18EI}$, A 點轉角為 $\theta''_A = \frac{4D_y L^2}{9EI}$

$\therefore C$ 點為滾支承

\therefore 在 C 點條件為 $v_C = \bar{v}_C + v'_C + v''_C = 0$

$$\Rightarrow -\frac{22wL^4}{24EI} + \frac{4C_y L^3}{9EI} + \frac{7D_y L^3}{18EI} = 0$$

$$\Rightarrow C_y = D_y = \frac{11wL}{10}$$

由 $\sum F_y = 0$ 可得 $A_y = B_y = \frac{1}{2}(3wL - 2 \cdot \frac{11wL}{10}) = \frac{2wL}{5}$

A 點轉角 $\theta_A = \bar{\theta}_A + \theta'_A + \theta''_A = -\frac{27wL^3}{24EI} + \frac{5C_y L^2}{9EI} + \frac{4D_y L^2}{9EI} = -\frac{wL^3}{40EI}$