國立臺灣海洋大學河海工程研究所2018材料力學2AB班第三次大考參考解答

日期：2018 年 06 月 27 日　　姓名：______________　　學號：______________

1. 一均質且為等向性之線彈性材料，其楊氏係數 $E = 50$ GPa ，蒲松比 $\nu = 0.25$ 。

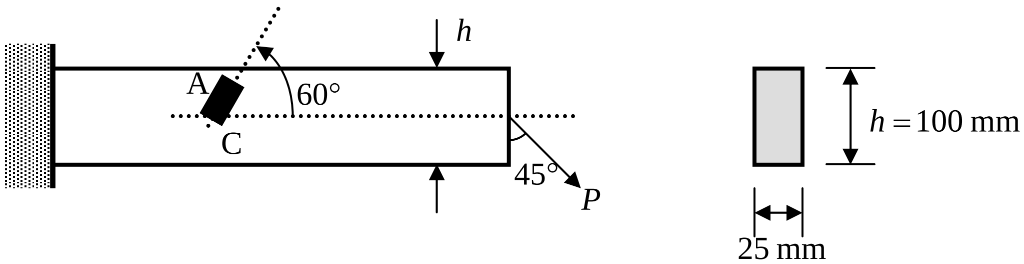
   在一個受外力的平面應力問題，某一點的應力大小為 $\sigma_{xx} = 140$ MPa ，

   $\sigma_{yy} = 20$ MPa ，$\tau_{xy} = 80$ MPa

   (1) 試畫出莫爾圓圖並求此點的主應力。(10%)

   (2) 求此點的剪應變 γ_{xy} 與正應變 ε_{xx} 。(10%)

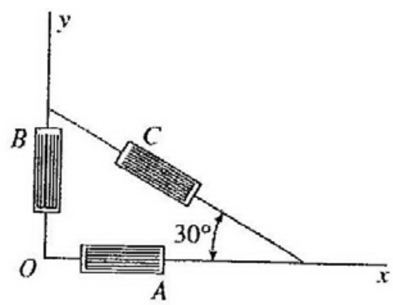
2. 如下圖所示之懸臂樑，其斷面為矩形(25 mm 寬、100 mm 高)。樑之彈性係數 $E = 200$ GPa，蒲松比 $\nu = 0.333$。此樑受一外力 P 作用於自由端之斷面形心上，力與斷面夾 45° 角，今一應變計 A 貼於 1/2 樑高的位置，如圖所示之 C 點並與其軸心現夾 60° 角。若量測值 $\varepsilon_A = -165\times10^{-6}$ ，求外力 P 之值及樑之最大剪應變 $\gamma_{\max}$ 。 (20%)



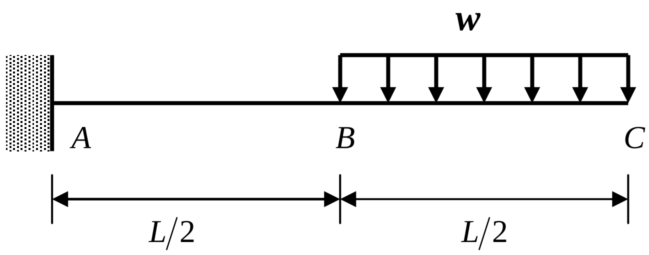
3. 一菊花座應變規貼附於樑上。各量規之讀數分別為 $\varepsilon_A = 1200\times10^{-6}$ 、

   $\varepsilon_B = 200\times10^{-6}$ 和 $\varepsilon_c = 200\times10^{-6}$ 。試求:

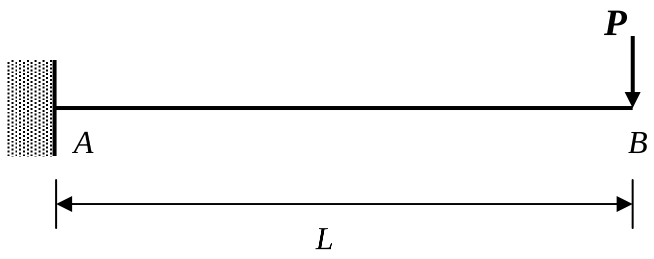
   (1)平面主應變 (5%)　　(2)最大平面剪應變 (5%)　　(3)平均正應變 (5%)



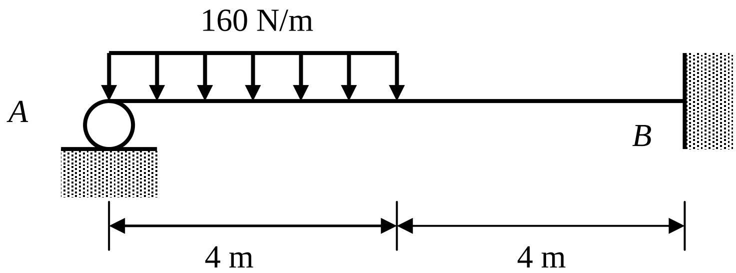
4. 試以積分法求下圖懸臂樑的彈性曲線方程式並計算 θ_C 與 v_C 為何? (20%)



5. 試以面積力矩法求下圖懸臂樑之 v_B 與 θ_B。 (10%)



6. 試求下圖 B 支承反力之大小與 θ_A。(15%)



7. (1) 上完了一學期的材料力學，對於這門課在學習上有何心得或感想? (5%)
   (2) 對於老師的教學方式或是要如何協助同學們學習好這門課有何建議? (5%)
   (有寫才有分)

(參考公式)

平面應力轉換方程式

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

平面應變轉換方程式

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

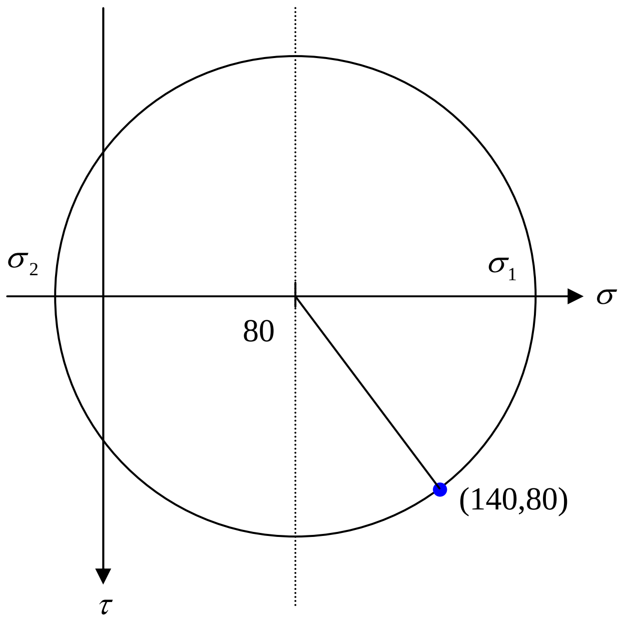
彎曲公式： $\sigma = -\dfrac{My}{I}$， **剪力公式：** $\tau = \dfrac{VQ}{It}$

參考解答:

1. 一均質且為等向性之線彈性材料，其楊氏係數 $E = 50$ GPa ，蒲松比 $\nu = 0.25$ 。在一個受外力的平面應力問題，某一點的應力大小為 $\sigma_{xx} = 140$ MPa ，$\sigma_{yy} = 20$ MPa ，$\tau_{xy} = 80$ MPa
   (1) 試畫出莫爾圓圖並求此點的主應力。(10%)
   (2) 求此點的剪應變 γ_{xy} 與正應變 ε_{xx} 。(10%)

(1) $\sigma_{xx} = 140$ MPa ，$\sigma_{yy} = 20$ MPa ，$\tau_{xy} = 80$ MPa

$$\Rightarrow \quad \sigma_{avg} = \frac{140+20}{2} = 80 \text{ (MPa)}$$



$$R = \sqrt{(140-80)^2 + 80^2} = 100$$

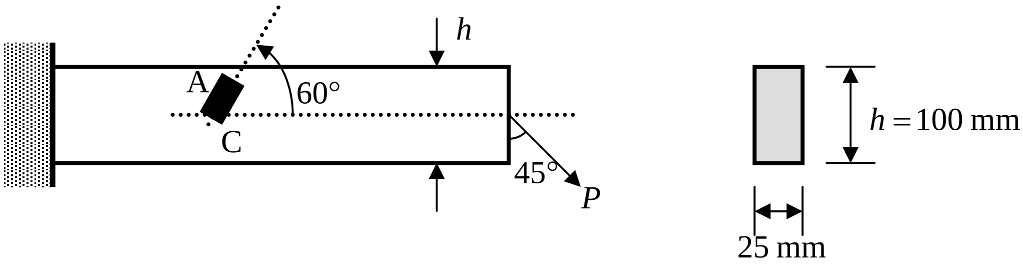
$$\sigma_1 = 80 + 100 = 180 \text{ (MPa)}$$
$$\sigma_2 = 80 - 100 = -20 \text{ (MPa)}$$

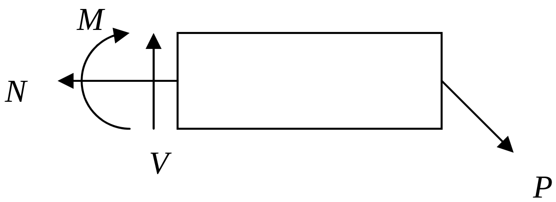
(2) $$G = \frac{E}{2(1+\nu)} = \frac{50}{2(1+0.25)} = 20 \text{ (GPa)}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{80 \cdot 10^6}{20 \cdot 10^9} = 4 \cdot 10^{-3}$$

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{yy} + \sigma_{zz}) = \frac{140 - 0.25(20+0)}{50 \cdot 10^3} = 2.7 \cdot 10^{-3}$$

2. 如下圖所示之懸臂樑，其斷面為矩形(25 mm 寬、100 mm 高)。樑之彈性係數 $E = 200$ GPa，蒲松比 $\nu = 0.333$。此樑受一外力 ***P*** 作用於自由端之斷面形心上，力與斷面夾 $45°$ 角，今一應變計 **A** 貼於 $1/2$ 樑高的位置，如圖所示之 **C** 點並與其軸心線夾 $60°$ 角。若量測值 $\varepsilon_A = -165\times10^{-6}$，求外力 ***P*** 之值及樑之最大剪應變 $\gamma_{\max}$。 **(20%)**





$$N = P\cdot\sin 45° = \frac{P}{\sqrt{2}} = 0.707P$$

$$V = P\cdot\cos 45° = \frac{P}{\sqrt{2}} = 0.707P$$

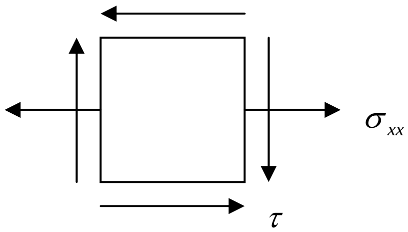
$\because$ A 貼於 $1/2$ 樑高的位置(C 點)，即為中性軸之位置，故彎曲應力為零

$$\therefore\ \sigma_x = \frac{N}{A} = \frac{0.707P}{100\cdot25\cdot10^{-6}} = (2.828\cdot10^2)P$$

$$\sigma_y = 0$$

$$\tau = \frac{VQ}{It} = \frac{V\cdot(b\cdot\frac{h}{2})\cdot\frac{h}{4}}{\frac{1}{12}bh^3\cdot b} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{V}{A}$$

$$= \frac{3}{2}\frac{0.707P}{100\cdot25\cdot10^{-6}} = (4.242\cdot10^2)P$$



由圖可知剪應力為負，故 $\tau_{xy} = -\tau$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y = \frac{(2.828\cdot 10^2)P}{200\cdot 10^9} = 1.414\cdot 10^{-9}P$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_x = -0.333\cdot\frac{(2.828\cdot 10^2)P}{200\cdot 10^9} = -0.471\cdot 10^{-9}P$$

$$G = \frac{E}{2(1+\nu)} = \frac{200}{2(1+0.333)} = 75\ \text{(GPa)}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-4.242\cdot 10^2 P}{75\cdot 10^9} = -5.656\cdot 10^{-9}P$$

平面應變轉換:

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\Rightarrow -165\cdot 10^{-6} = (\frac{1.414-0.471}{2} + \frac{1.414+0.471}{2}\cos 120^\circ + \frac{-5.656}{2}\sin 120^\circ)\cdot 10^{-9}P$$

$$\Rightarrow P = 67378\ \text{(N)} = 67.378\ \text{(kN)}$$

$$\therefore\ \varepsilon_x = 1.414\cdot 10^{-9}P = 95.273\cdot 10^{-6}$$

$$\varepsilon_y = -0.47\cdot 10^{-9}P = -31.735\cdot 10^{-6}$$

$$\gamma_{xy} = -5.656\cdot 10^{-9}P = -374.959\cdot 10^{-6}$$

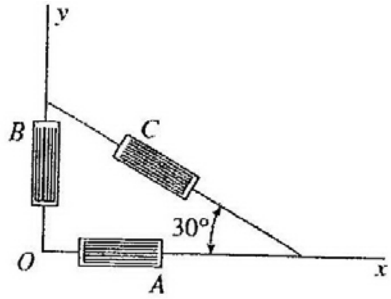
$$\frac{\gamma_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{95.273+31.735}{2}\right)^2 + \left(\frac{374.959}{2}\right)^2}\cdot 10^{-6}$$

$$\Rightarrow \frac{\gamma_{\max}}{2} = 197.943\cdot 10^{-6}$$

$$\Rightarrow \gamma_{\max} = 395.886\cdot 10^{-6}$$

3. 一菊花座應變規貼附於樑上。各量規之讀數分別為 $\varepsilon_A = 1200\times10^{-6}$ **、** $\varepsilon_B = 200\times10^{-6}$ **和** $\varepsilon_c = 200\times10^{-6}$ **。試求:**

(1)平面主應變 (5%)　　(2)最大平面剪應變 (5%)　　(3)平均正應變 (5%)



$$\varepsilon_{x'} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_A = 1200\times10^{-6} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 0^\circ + \frac{\gamma_{xy}}{2}\sin 0^\circ$$

$$\Rightarrow \varepsilon_x = 1200\times10^{-6}$$

$$\varepsilon_B = 200\times10^{-6} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 180^\circ + \frac{\gamma_{xy}}{2}\sin 180^\circ$$

$$\Rightarrow \varepsilon_y = 200\times10^{-6}$$

$$\varepsilon_C = 200\times10^{-6} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 300^\circ + \frac{\gamma_{xy}}{2}\sin 300^\circ$$

$$\Rightarrow \varepsilon_y = 1000\sqrt{3}\times10^{-6} = 1732\times10^{-6}$$

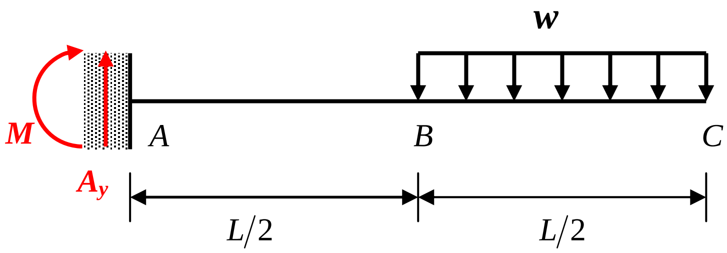
(1) $$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{(\frac{\varepsilon_x-\varepsilon_y}{2})^2 + (\frac{\gamma_{xy}}{2})^2} = 1700\times10^{-6} \text{ and } -300\times10^{-6}$$

(2) $$\frac{\gamma_{\max}}{2} = \sqrt{(\frac{\varepsilon_x-\varepsilon_y}{2})^2 + (\frac{\gamma_{xy}}{2})^2} = 1000\times10^{-6}$$

$$\Rightarrow \gamma_{\max} = 2000\times10^{-6}$$

(3) $$\varepsilon_{avg} = \frac{\varepsilon_1+\varepsilon_2}{2} = 700\times10^{-6}$$

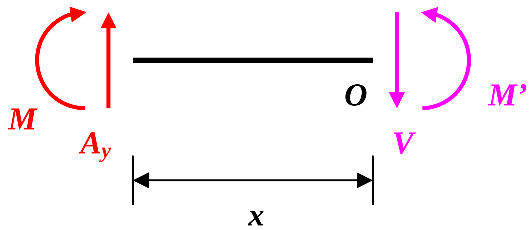
4. 試以積分法求下圖懸臂樑的彈性曲線方程式並計算 θ_C 與 v_C 為何? (20%)



$$\sum F_y = 0 \quad \Rightarrow A_y = \frac{wL}{2}$$

$$\sum M_A = 0 \quad \Rightarrow M = -\frac{3wL^2}{8}$$

(1) 當 $0 \le x \le \frac{L}{2}$



$$\sum M_O = 0 \quad \Rightarrow M' = \frac{wL}{2}x - \frac{3wL^2}{8}$$

$$\therefore \quad \frac{d^2v}{dx^2} = \frac{M'}{EI} = \frac{wL}{2EI}x - \frac{3wL^2}{8EI}$$

$$\Rightarrow \theta_1 = \frac{dv}{dx} = \frac{wL}{4EI}x^2 - \frac{3wL^2}{8EI}x + C_1$$

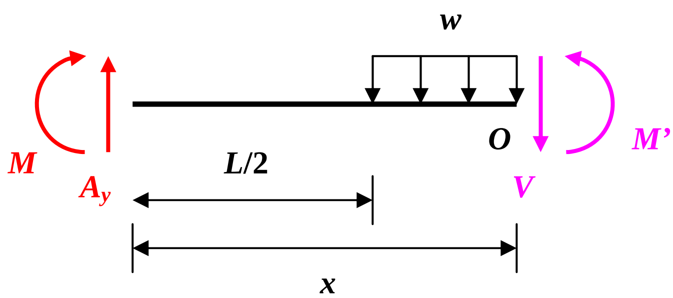
$$\Rightarrow v_1 = \frac{wL}{12EI}x^3 - \frac{3wL^2}{16EI}x^2 + C_1x + C_2$$

又 $\theta_1(0) = 0 \quad \Rightarrow C_1 = 0$

$v_1(0) = 0 \quad \Rightarrow C_2 = 0$

$$\therefore v_1(x) = \frac{wL}{12EI}x^3 - \frac{3wL^2}{16EI}x^2, \qquad \theta_1(x) = \frac{wL}{4EI}x^2 - \frac{3wL^2}{8EI}x$$

(2) 當 $\frac{L}{2} \le x \le L$



$$\sum M_O = 0 \quad \Rightarrow M' = -\frac{w}{2}x^2 + wLx - \frac{wL^2}{2}$$

$$\therefore \ \frac{d^2v}{dx^2} = \frac{M'}{EI} = -\frac{w}{2EI}x^2 + \frac{wL}{EI}x - \frac{wL^2}{2EI}$$

$$\Rightarrow \theta_2 = \frac{dv}{dx} = -\frac{w}{6EI}x^3 + \frac{wL}{2EI}x^2 - \frac{wL^2}{2EI}x + C_3$$

$$\Rightarrow v_2 = -\frac{w}{24EI}x^4 + \frac{wL}{6EI}x^3 - \frac{wL^2}{4EI}x^2 + C_3x + C_4$$

又 $\theta_1(\frac{L}{2}) = \theta_2(\frac{L}{2})$

$$\Rightarrow -\frac{wL^3}{8EI} = -\frac{wL^3}{48EI} + \frac{wL^3}{8EI} - \frac{wL^3}{4EI} + C_3 \quad \Rightarrow C_3 = \frac{wL^3}{48EI}$$

$$v_1(\frac{L}{2}) = v_2(\frac{L}{2})$$

$$\Rightarrow -\frac{7wL^4}{192EI} = -\frac{wL^4}{384EI} + \frac{wL^4}{48EI} - \frac{wL^4}{16EI} + \frac{wL^4}{96EI} + C_4 \Rightarrow C_4 = -\frac{wL^4}{384EI}$$

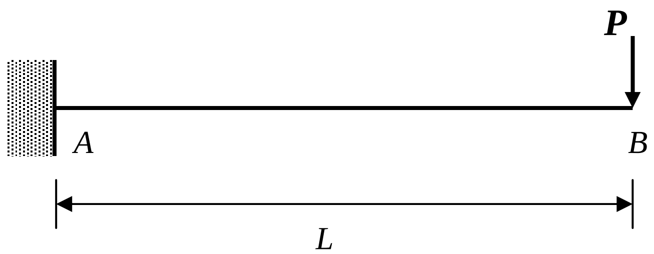
$$\therefore \theta_2(x) = \frac{dv}{dx} = -\frac{w}{6EI}x^3 + \frac{wL}{2EI}x^2 - \frac{wL^2}{2EI}x + \frac{wL^3}{48EI}$$

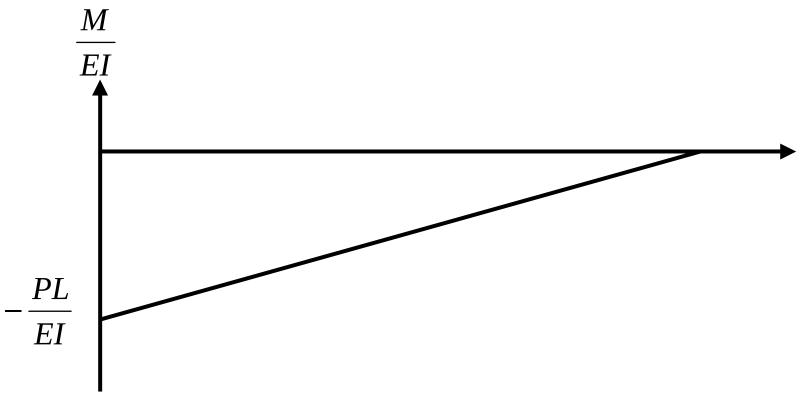
$$v_2(x) = -\frac{w}{24EI}x^4 + \frac{wL}{6EI}x^3 - \frac{wL^2}{4EI}x^2 + \frac{wL^3}{48EI}x - \frac{wL^4}{384EI}$$

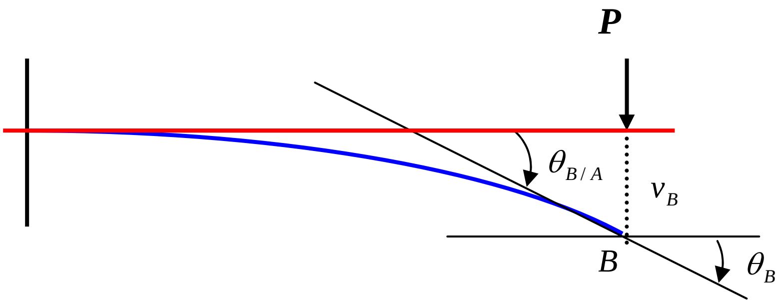
$$\theta_C = \theta_2(L) = -\frac{wL^3}{6EI} + \frac{wL^3}{2EI} - \frac{wL^3}{2EI} + \frac{wL^3}{48EI} = -\frac{7wL^3}{48EI}$$

$$v_C = v_2(L) = -\frac{wL^4}{24EI} + \frac{wL^4}{6EI} - \frac{wL^4}{4EI} + \frac{wL^4}{48EI} - \frac{wL^4}{384EI} = -\frac{41wL^4}{384EI}$$

5. 試以面積力矩法求下圖懸臂樑之 v_B 與 θ_B。 (10%)



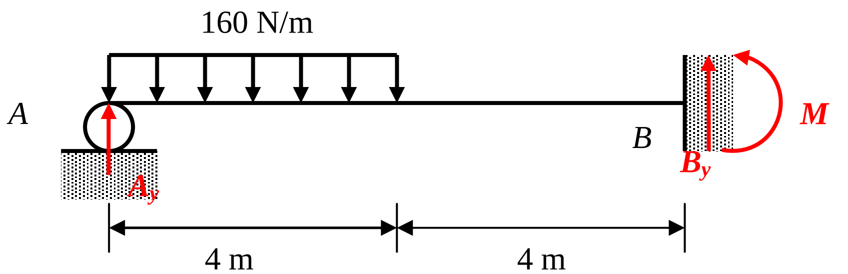




$$\theta_B = \theta_{B/A} = \frac{1}{2} \cdot L \cdot \left(-\frac{PL}{EI}\right) = -\frac{PL^2}{2EI}$$

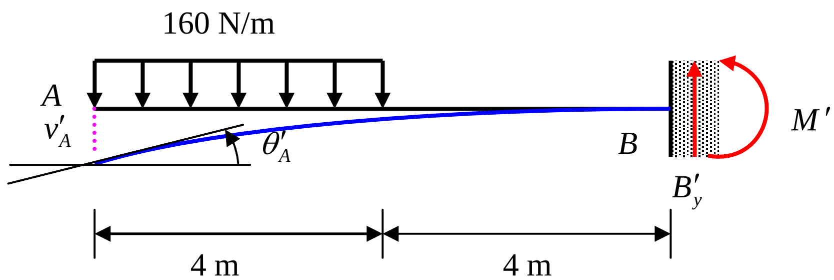
$$v_B = t_{B/A} = \frac{1}{2} \cdot L \cdot \left(-\frac{PL}{EI}\right) \cdot \frac{2L}{3} = -\frac{PL^3}{3EI}$$

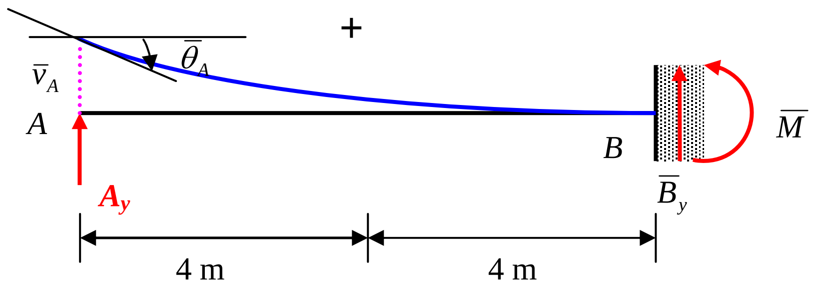
6. 試求下圖 *B* 支承反力之大小與θ_A。(15%)



此為 1 度靜不定問題，可將 A_y 視為贅力，

並透過疊加法將問題分解為兩部份





並且在 A 處有 $v_A = 0$

第一部分由第四題結果可知： $\theta'_A = \dfrac{7wL^3}{48EI} = \dfrac{35840}{3EI}$

$$v'_A = -\frac{41wL^4}{384EI} = -\frac{209920}{3EI}$$

第二部分由第五題結果可知： $\bar{\theta}_A = -\dfrac{32A_y}{EI}$

$$\bar{v}_A = \frac{A_yL^3}{3EI} = \frac{512A_y}{3EI}$$

又 $v_A = v'_A + \bar{v}_A = 0 \quad \Rightarrow -\dfrac{209920}{3EI} + \dfrac{512A_y}{3EI} = 0 \quad \Rightarrow A_y = 410\ (\text{N})$

$\therefore \quad M = -(160\cdot 4)\cdot 6 + 410\cdot 8 = -560\ (\text{N}\cdot\text{m})$

$$B_y = 640 - 410 = 230\ (\text{N})$$

$$\theta_A = \theta'_A + \bar{\theta}_A = \frac{35840}{3EI} - \frac{32\cdot 410}{EI} = -\frac{3520}{3EI}$$