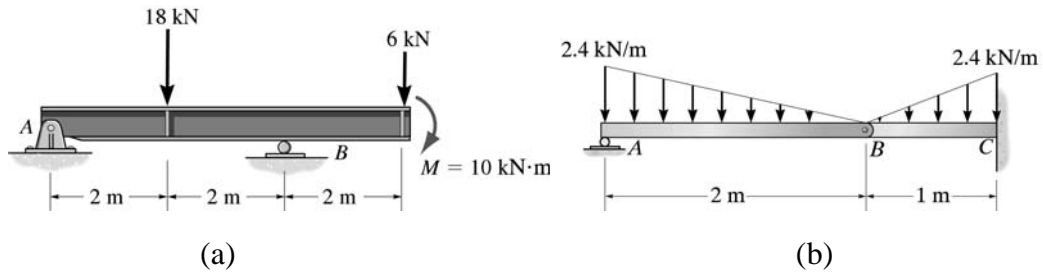
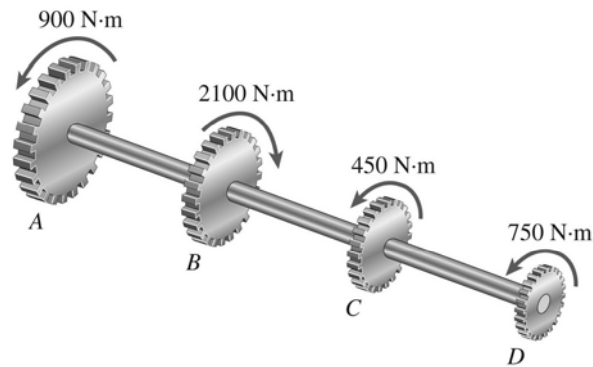


1. 請畫出下面各樑的剪力圖和彎矩圖。(20%)

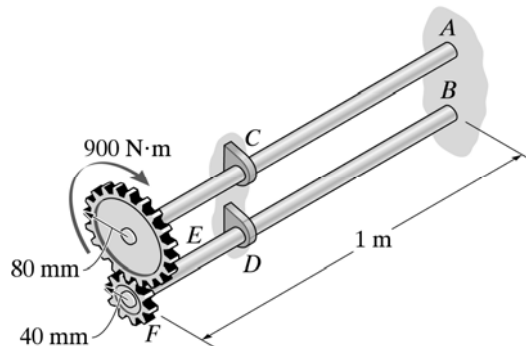


2. 四組齒輪接於圓軸，並傳遞圖示的扭矩。圓軸的容許剪應力為 70 MPa。

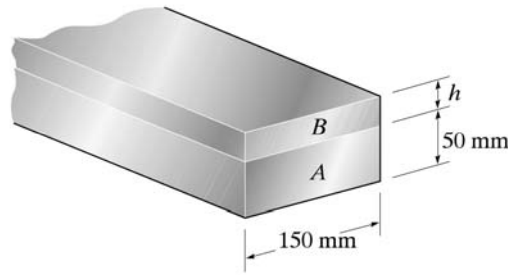
- (a) 假設圓軸為實心橫截面，求該軸所需的直徑 d 。(10%)
 (b) 假設圓軸為內徑為 40 mm 的中空軸，求所需外徑 d 。(10%)



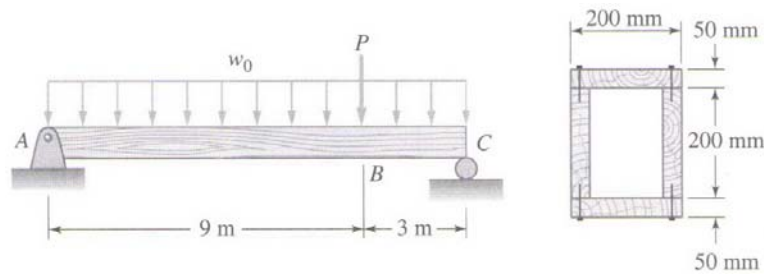
3. 兩根 1 m 長的軸由 2014-T6 鋁合金製成。各直徑為 30 mm 且由固定在各軸端的齒輪相連接。各軸的另一端則附在固定支承 A 及 B 上。此外並由 C 及 D 軸承支撐，允許軸沿軸心自由轉動。若一扭矩 900 N·m 作用在上部齒輪，如圖所示，試求各軸的最大剪應力。(20%)



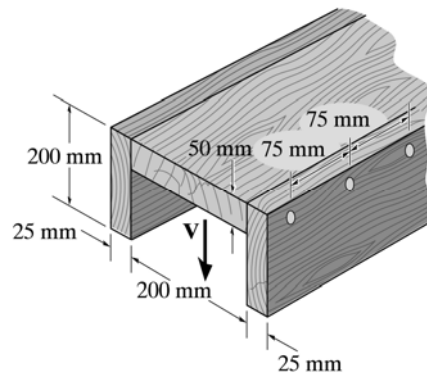
4. 複合樑係由 6061-T6 鋁合金(A)及 C83400 紅黃銅(B)所製。
- 試求欲使樑之中性軸位於兩金屬的接縫處時的尺寸 h 。(10%)
 - 若鋁合金的容許應力為 $(\sigma_{\text{allow}})_{\text{al}} = 128 \text{ MPa}$ ，黃銅的為 $(\sigma_{\text{allow}})_{\text{br}} = 35 \text{ MPa}$ ，則樑可支承的最大彎矩為何？ ($E_{\text{al}} = 68.9 \text{ GPa}$ ， $E_{\text{br}} = 101 \text{ GPa}$) (10%)



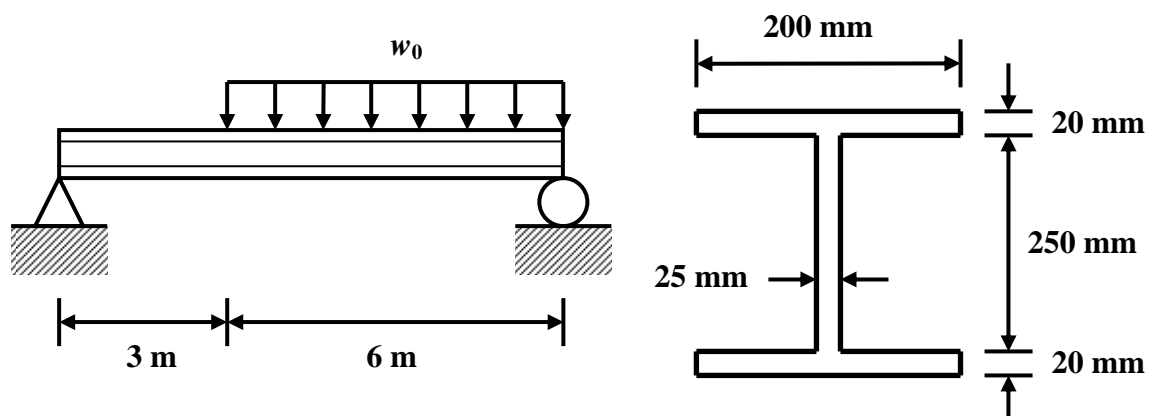
5. 一箱形樑如圖所示，由四片 $50 \text{ mm} \times 200 \text{ mm}$ 之木板釘組而成。
- 試求截面對中性軸的慣性矩。(10%)
 - 已知 $w_0 = 300 \text{ N/m}$ ，若彎曲應力限 8.4 MPa 以內，求最大容許力 P 。(10%)



6. 樑承受一剪力 $V = 2 \text{ kN}$ 。若樑每邊之釘距為 75 mm ，試求各釘中所產生的平均剪應力。各釘直徑均為 4 mm 。(20%)



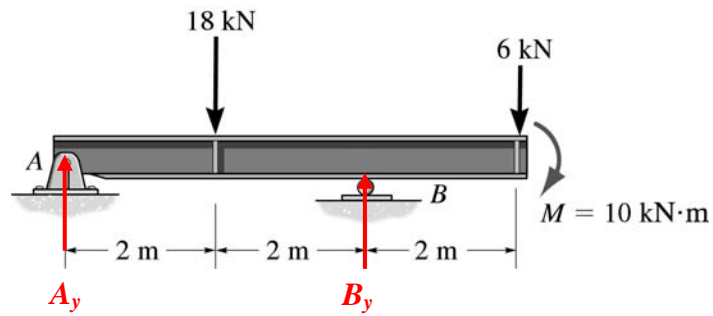
7. 樑之容許彎曲應力為 10 MPa ，容許剪應力為 1.4 MPa ，求容許最大負荷強度 $w_0 = ?$ (20%)



參考解答:

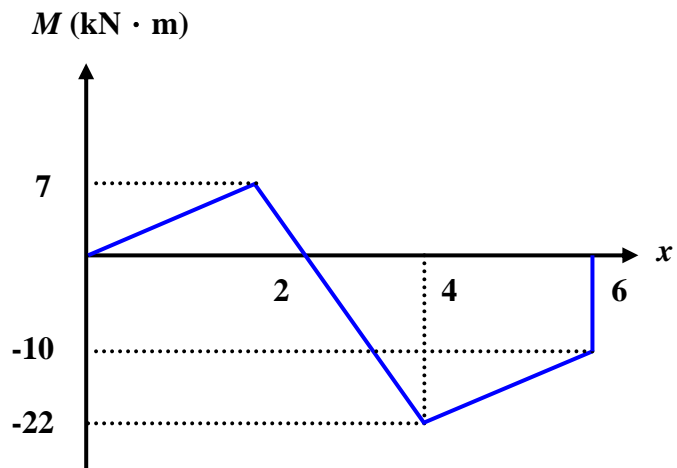
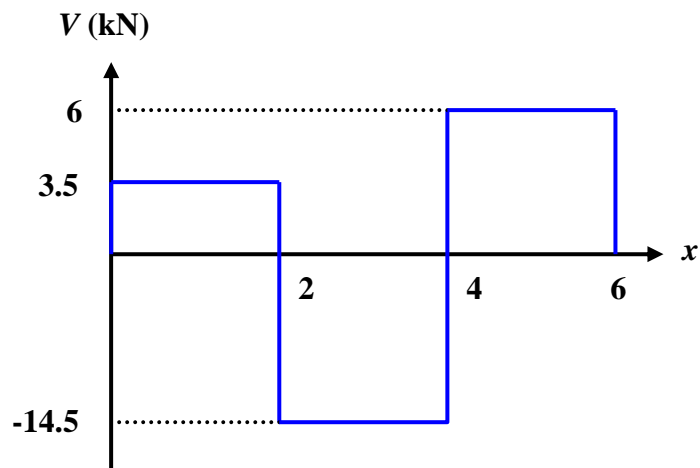
1. 請畫出下面各樑的的剪力圖和彎矩圖。(20%)

(a)

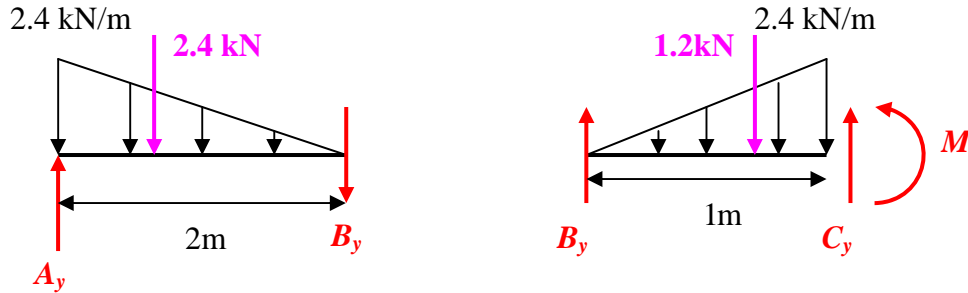
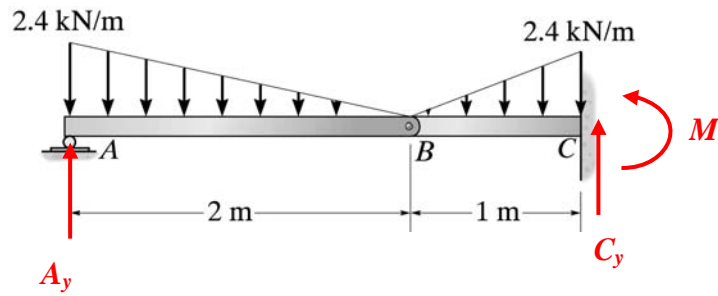


$$\sum M_A = 0 \Rightarrow 18 \cdot 2 - B_y \cdot 4 + 6 \cdot 6 + 10 = 0 \Rightarrow B_y = 20.5 \text{ (kN)}$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - 24 = 0 \Rightarrow A_y = 3.5 \text{ (kN)}$$



(b)

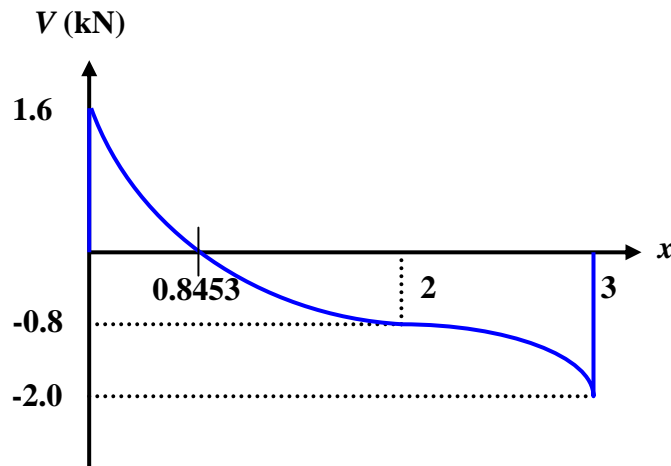


$$\sum M_A = 0 \Rightarrow 2.4 \cdot \frac{2}{3} + B_y \cdot 2 = 0 \Rightarrow B_y = -0.8 \text{ (kN)}$$

$$\sum F_y = 0 \Rightarrow A_y - B_y - 2.4 = 0 \Rightarrow A_y = 1.6 \text{ (kN)}$$

$$\sum F_y = 0 \Rightarrow C_y + B_y - 1.2 = 0 \Rightarrow C_y = 2.0 \text{ (kN)}$$

$$\sum M_C = 0 \Rightarrow M + 1.2 \cdot \frac{1}{3} - B_y \cdot 1 = 0 \Rightarrow M = -1.2 \text{ (kN)}$$



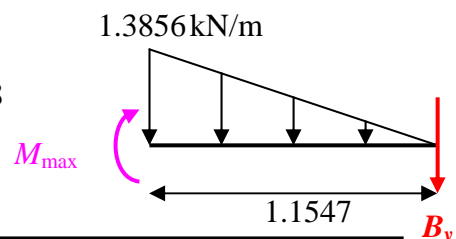
剪力為零的點，最大彎矩值

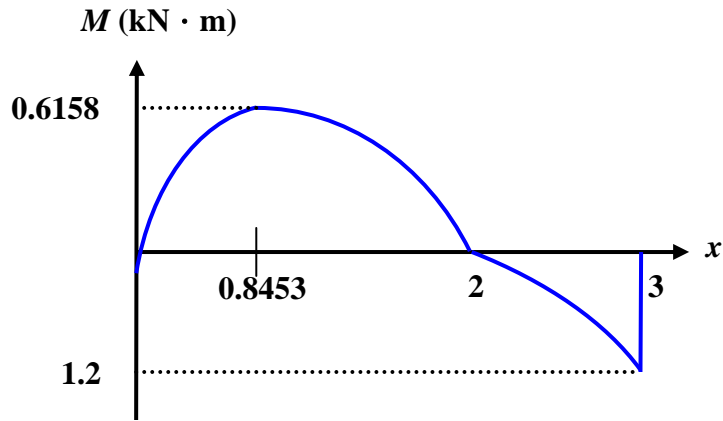
$$\frac{1}{2}x \cdot (1.2x) = 0.8 \Rightarrow x = \sqrt{\frac{4}{3}} = 1.1547$$

∴ 最大彎矩值發生在由內銷支承向左 1.1547 處，即 A 點右邊 0.8453 處

最大彎矩值為

$$M_{\max} = \frac{1}{2} \cdot 1.1547 \cdot 1.3856 \cdot \left(\frac{2}{3} \cdot 1.1547\right) = 0.6158$$

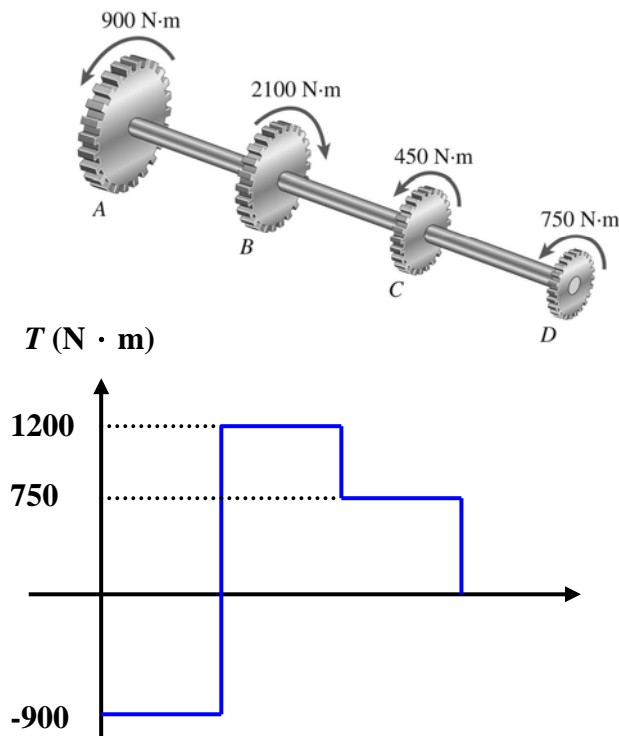




2. 四組齒輪接於圓軸，並傳遞圖示的扭矩。圓軸的容許剪應力為 70 MPa。

(a) 假設圓軸為實心橫截面，求該軸所需的直徑 d 。(10%)

(b) 假設圓軸為內徑為 40 mm 的中空軸，求所需外徑 d 。(10%)



$$(a) \tau_{\max} = \frac{T \cdot c}{J} = \frac{16T}{\pi d^3}$$

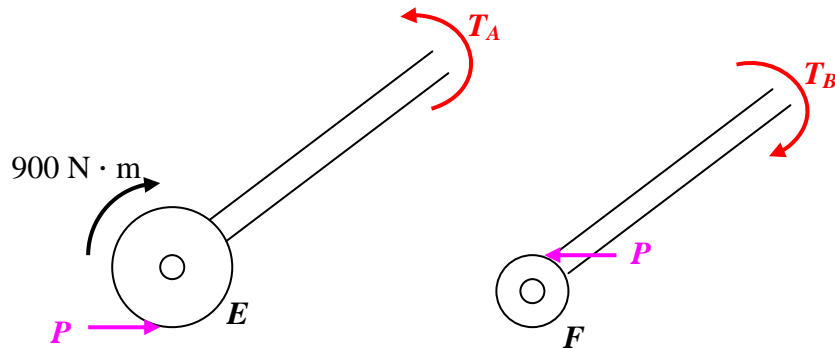
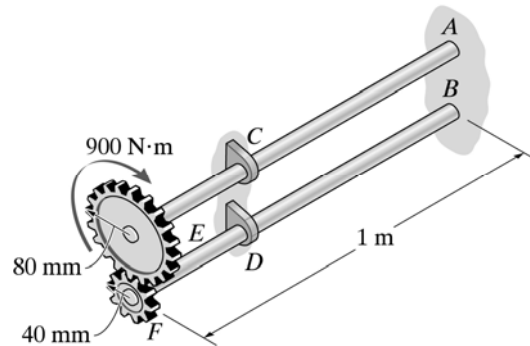
$$70 \cdot 10^6 = \frac{16 \cdot 1200}{\pi d^3} \Rightarrow d = 0.04436 \text{ (m)} = 44.36 \text{ (mm)}$$

$$(b) \tau_{\max} = \frac{T \cdot c}{J} = \frac{T \cdot \frac{d_o}{2}}{\frac{\pi}{2} \left[\left(\frac{d_o}{2} \right)^4 - \left(\frac{d_i}{2} \right)^4 \right]} = \frac{16T \cdot d_o}{\pi [(d_o)^4 - (d_i)^4]}$$

(d_o : 外直徑, d_i : 內直徑)

$$70 \cdot 10^6 = \frac{16 \cdot 1200 \cdot d}{\pi (d^4 - 0.04^4)} \Rightarrow d = 0.05155 \text{ (m)} = 51.55 \text{ (mm)}$$

3. 兩根 1 m 長的軸由 2014-T6 鋁合金製成。各直徑為 30 mm 且由固定在各軸端的齒輪相連接。各軸的另一端則附在固定支承 A 及 B 上。此外並由 C 及 D 軸承支撐，允許軸沿軸心自由轉動。若一扭矩 900 N·m 作用在上部齒輪，如圖所示，試求各軸的最大剪應力。(20%)



$$\sum M_F = 0 \Rightarrow P \cdot r_F = T_B \Rightarrow T_B = 0.04P$$

$$\sum M_E = 0 \Rightarrow 900 - P \cdot r_E - T_A = 0 \Rightarrow T_A = 900 - 0.08P = 900 - 2T_B$$

又 E 齒輪與 F 齒輪所旋轉弧長相同

$$\therefore \phi_E \cdot r_E = \phi_F \cdot r_F \Rightarrow \phi_E = 0.5 \phi_F$$

$$\Rightarrow \frac{T_A L}{JG} = 0.5 \frac{T_B L}{JG} \Rightarrow T_A = 0.5 T_B$$

$$\text{故可知 } 2.5 T_B = 900 \Rightarrow T_B = 360 \text{ (N} \cdot \text{m)}$$

$$\Rightarrow T_A = 180 \text{ (N} \cdot \text{m)}$$

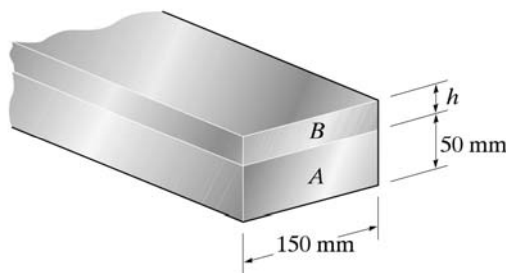
$$(\tau_{AE})_{\max} = \frac{T_A \cdot c}{J} = \frac{180 \cdot 0.015}{\frac{\pi}{2} \cdot (0.015)^4} = 33.95 \cdot 10^6 \text{ (Pa)} = 33.95 \text{ (MPa)}$$

$$(\tau_{BF})_{\max} = \frac{T_B \cdot c}{J} = \frac{360 \cdot 0.015}{\frac{\pi}{2} \cdot (0.015)^4} = 67.91 \cdot 10^6 \text{ (Pa)} = 67.91 \text{ (MPa)}$$

4. 複合樑係由 6061-T6 鋁合金(A)及 C83400 紅黃銅(B)所製。

(1) 試求欲使樑之中性軸位於兩金屬的接縫處時的尺寸 h 。(10%)

(2) 若鋁合金的容許應力為 $(\sigma_{\text{allow}})_{\text{al}} = 128 \text{ MPa}$ ，黃銅的為 $(\sigma_{\text{allow}})_{\text{br}} = 35 \text{ MPa}$ ，則樑可支承的最大彎矩為何？ ($E_{\text{al}} = 68.9 \text{ GPa}$ ， $E_{\text{br}} = 101 \text{ GPa}$) (10%)

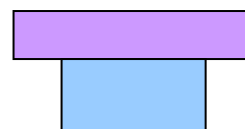


$$(1) \quad n = \frac{E_{\text{al}}}{E_{\text{br}}} = \frac{68.9}{101} = 0.6822$$

$$b'_{\text{br}} = n b_{\text{br}} = 0.6822 \cdot 150 = 102.33 \text{ (mm)}$$

$$\bar{y}' = \frac{\sum A\bar{y}}{\sum A} \Rightarrow 50 = \frac{(102.33 \cdot 50) \cdot 25 + (150 \cdot h) \cdot (50 + \frac{h}{2})}{102.33 \cdot 50 + 150 \cdot h}$$

$$\Rightarrow h = 41.30 \text{ (mm)}$$



$$I = \frac{1}{12} (102.33) \cdot (50)^3 + (102.33 \cdot 50) \cdot (50 - 25)^2$$

$$+ \frac{1}{12} (150) \cdot (41.30)^3 + (150 \cdot 41.30) \cdot (\frac{41.30}{2})^2$$

$$\Rightarrow I = 7.7851 \cdot 10^6 \text{ (mm}^4\text{)} = 7.7851 \cdot 10^{-6} \text{ (m}^4\text{)}$$

$$(\sigma_{\text{allow}})_{\text{br}} = \frac{M \cdot c}{I} \Rightarrow 35 \cdot 10^6 = \frac{M \cdot 0.0413}{7.7851 \cdot 10^{-6}}$$

$$\Rightarrow M = 6598 \text{ (N} \cdot \text{m)} \approx 6.60 \text{ (kN} \cdot \text{m)}$$

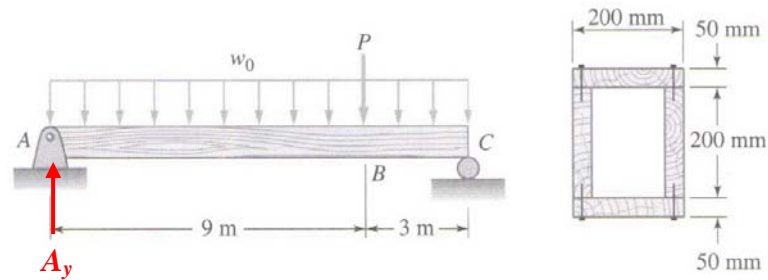
$$(\sigma_{\text{allow}})_{\text{al}} = n \frac{M \cdot c}{I} \Rightarrow 128 \cdot 10^6 = 0.6822 \cdot \frac{M \cdot 0.5}{7.7851 \cdot 10^{-6}}$$

$$\Rightarrow M = 29215 \text{ (N} \cdot \text{m)} \approx 29.22 \text{ (kN} \cdot \text{m)}$$

5. 一箱形樑如圖所示，由四片 50 mm × 200 mm 之木板釘組而成。

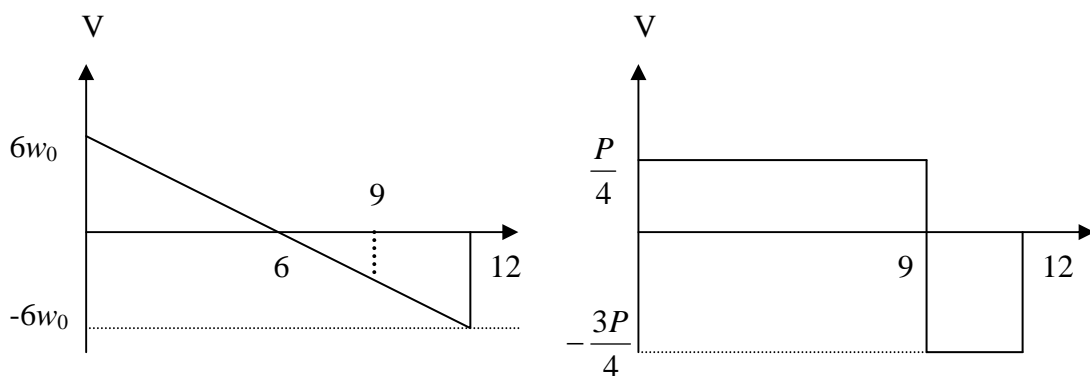
(a) 試求截面對中性軸的慣性矩。(10%)

(b) 已知 $w_0 = 300 \text{ N/m}$ ，若彎曲應力限 8.4 MPa 以內，求最大容許力 P 。(10%)



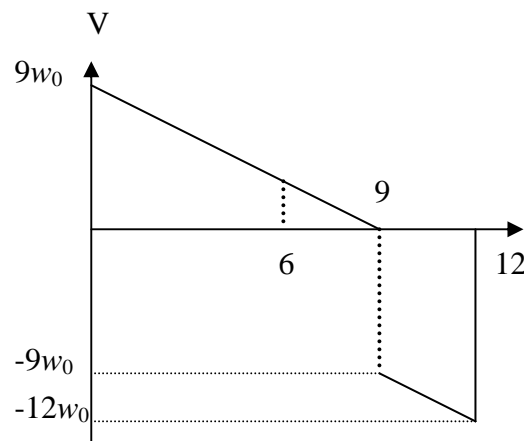
$$(a) I = \frac{1}{12} 200 \cdot 300^3 - \frac{1}{12} 100 \cdot 200^3 = 383.33 \cdot 10^6 \text{ (mm}^4\text{)} = 383.33 \cdot 10^{-6} \text{ (m}^4\text{)}$$

(b) 彎矩最大值發生在剪力為零處



可看出剪力為零會發生在 $x=9$ 之處

$$\text{此時 } \frac{P}{4} - 3w_0 = 0 \Rightarrow P = 12w_0$$



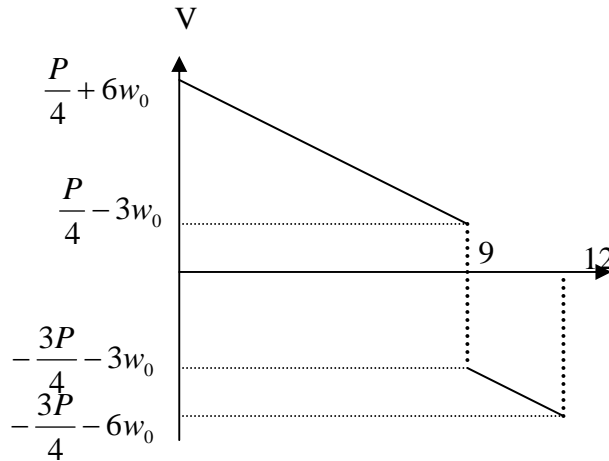
$$\therefore M_{\max} = \frac{1}{2} \cdot 9w_0 \cdot 9 = 40.5w_0$$

$$\text{由 } \sigma_{\max} = \frac{M_{\max} c}{I} \text{ 可知 } 8.4 \cdot 10^6 = \frac{M_{\max} \cdot 0.15}{383.33 \cdot 10^{-6}} \Rightarrow M_{\max} = 24166.48 \text{ (N} \cdot \text{m)}$$

$$\therefore 40.5w_0 = 24166.48 \Rightarrow w_0 = 530 \text{ (N/m)}$$

顯然 w_0 已超出題目所給定之值 $w_0 = 300 \text{ (N/m)}$

所以剪力圖應該如下

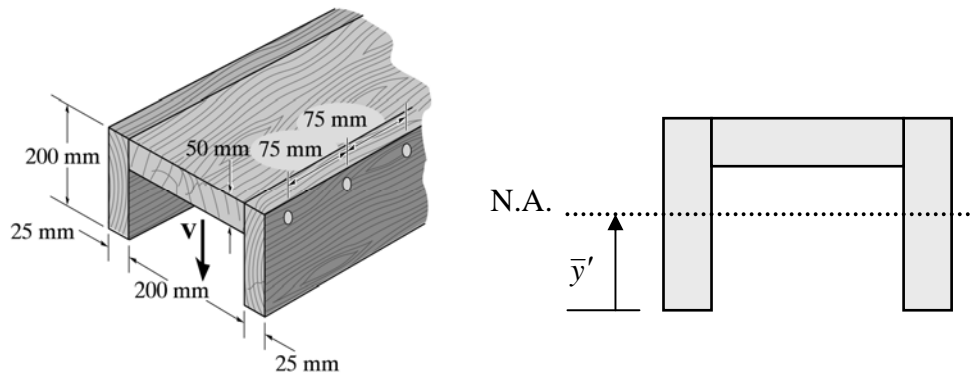


$$\therefore M_{\max} = \frac{1}{2} \cdot \left(\frac{P}{4} + 6w_0 + \frac{P}{4} - 3w_0 \right) \cdot 9 = \frac{9P}{4} + \frac{27w_0}{2}$$

由 $M_{\max} = 24166.48 \text{ (N}\cdot\text{m)}$ 可得

$$\frac{9P}{4} + \frac{27 \cdot 300}{2} = 24166.48 \quad \Rightarrow P = 8940.66 \text{ (N)}$$

6. 樑承受一剪力 $V = 2 \text{ kN}$ 。若樑每邊之釘距為 75 mm ，試求各釘中所產生的平均剪應力。各釘直徑均為 4 mm 。(20%)



$$\bar{y}' = \frac{\sum A\bar{y}}{\sum A} \quad \Rightarrow \bar{y}' = \frac{(200 \cdot 25) \cdot 100 \cdot 2 + (200 \cdot 50) \cdot (200 - 25)}{(200 \cdot 25) \cdot 2 + (200 \cdot 50)} = 137.5 \text{ (mm)}$$

$$I = \left[\frac{1}{12} (25) \cdot (200)^3 + (25 \cdot 200) \cdot (137.5 - 100)^2 \right] \cdot 2$$

$$+ \frac{1}{12} (200) \cdot (50)^3 + (200 \cdot 50) \cdot (175 - 137.5)^2$$

$$\Rightarrow I = 63.5417 \cdot 10^6 \text{ (mm}^4) = 63.5417 \cdot 10^{-6} \text{ (m}^4)$$

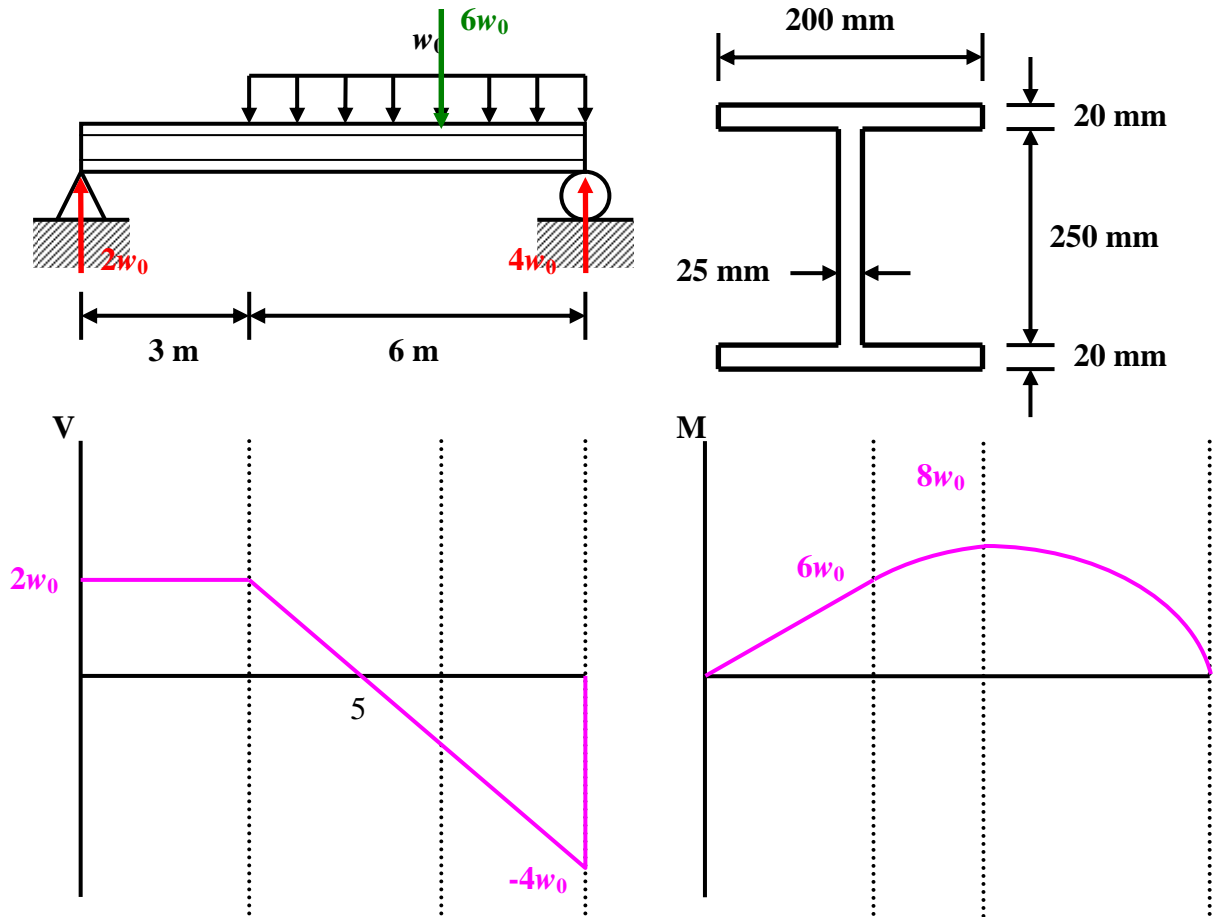
$$Q = (200 \cdot 50) \cdot (175 - 137.5) = 375000 \text{ (mm}^3) = 0.375 \cdot 10^{-3} \text{ (m}^3)$$

$$q = \frac{VQ}{I} = \frac{2 \cdot 10^3 \cdot 0.375 \cdot 10^{-3}}{63.5417 \cdot 10^{-6}} = 11.80 \cdot 10^3$$

$$\text{又 } \frac{F}{s} = \frac{q}{2} \Rightarrow \frac{F}{0.075} = \frac{11.80 \cdot 10^3}{2} \Rightarrow F = 442.5 \text{ (N)}$$

$$\tau = \frac{F}{A} \Rightarrow \tau = \frac{442.5}{\pi \cdot (0.002)^2} = 35.21 \cdot 10^6 \text{ (Pa)} = 35.21 \text{ (MPa)}$$

7. 樑之容許彎曲應力為 10 MPa，容許剪應力為 1.4 MPa，求容許最大負荷強度 $w_0 = ?$ (20%)



彎曲應力公式： $\sigma = \frac{Mc}{I}$ 剪力公式： $\tau = \frac{VQ}{It}$

$$I = \frac{1}{12}(200 \cdot 290^3) - \frac{1}{12}(175 \cdot 250^3) = 178.62 \cdot 10^6 \text{ (mm}^4) = 178.62 \cdot 10^{-6} \text{ (m}^4)$$

$$Q = (200 \cdot 20) \cdot 135 + (25 \cdot 125) \cdot 62.5 = 735312.5 \text{ (mm}^3) = 0.7353 \cdot 10^{-3} \text{ (m}^3)$$

$$\sigma = \frac{Mc}{I} \Rightarrow 10 \cdot 10^6 = \frac{8w_0 \cdot 0.145}{178.62 \cdot 10^{-6}} \Rightarrow w_0 = 1539.83 \text{ (N/m)}$$

$$\tau = \frac{VQ}{It} \Rightarrow 1.4 \cdot 10^6 = \frac{4w_0 \cdot 0.7353 \cdot 10^{-3}}{178.62 \cdot 10^{-6} \cdot 0.025} \Rightarrow w_0 = 2125.56 \text{ (N/m)}$$

\therefore 容許最大負荷強度 w_0 為 1539.83 N/m