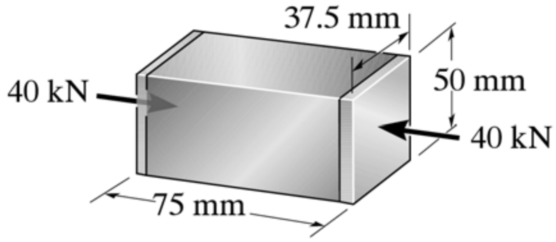
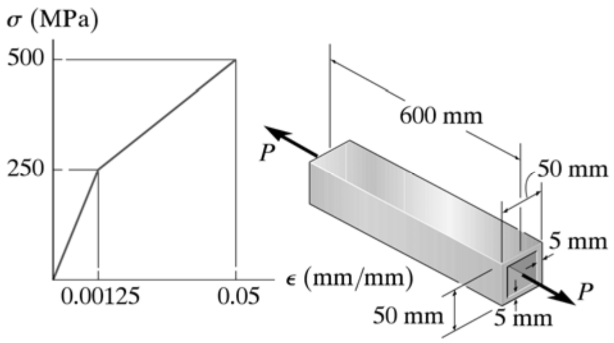
國立臺灣海洋大學河海工程學系 2018 材料力學 2AB 班第一次大考參考解答

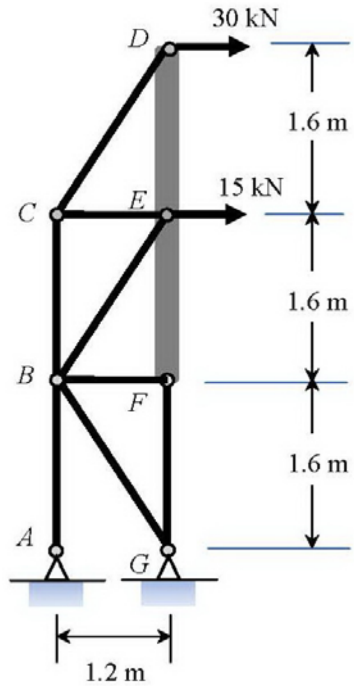
日期：2018 年 05 月 03 日　　姓名：________　　學號：________

1. 鋁塊有矩形截面，受一軸向壓縮力 40 kN 作用。若 37.5 mm 的邊長度為 37.5033 mm，試求蒲松比及 50 mm 邊的新長度。 $E_{al} = 70$ GPa 。 (20%)



2. (1) 當一軸向負載 $P = 100$ kN 施加在一方形中空棒，試求其伸長量。 (7%)
   (2) 若此負載增加至 $P = 360$ kN 然後釋放，試求其永久伸長量。 (8%)
   (棒材之應力 − 應變近似圖如下左圖。)





3. *DEF* 為一剛性桿件（rigid bar）且由一桁架系統支撐（如上右圖）。若桁架中各桿件之剪彈性模數（shear modulus） $G = 77.2$ GPa，柏松比（Poisson's ratio） $\nu = 0.3$，且容許正向應力（allowable normal stress）為 120 MPa 。
   (1) 請求出點 *A* 及 *G* 之反力。 (5%)
   (2) 請問在考慮容許正向應力條件下，*BG* 桿之最小斷面積。 (10%)
   (3) 若 *FG* 為直徑 40 mm 之實心桿件，請問 *FG* 桿之變形量？ (10%)

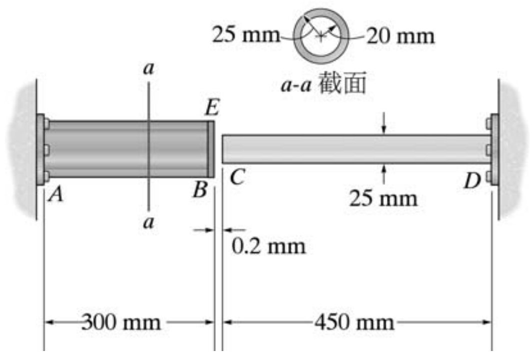
4. AM1004-T61 鎂合金管 *AB* 與剛性板 *E* 固定在一定，*E* 端和 6016-T6 鋁合金實心圓桿 *CD* 的 *C* 端間的間隙為 0.2 mm，其溫度為 30°C。若溫度上升至 80°C 時，剛性端蓋厚可忽略

(1) 試求管與桿件中的正應力。 (10%)

(2) 若管或桿件不發生降伏的情形下，溫度最高可上升至多少 °C 。 (10%)

(鎂: $\alpha_{ma} = 26 \cdot 10^{-6} / °C$, , $E_{ma} = 44.7$ GPa , $(\sigma_Y)_{ma} = 152$ MPa)

(鋁: $\alpha_{al} = 24 \cdot 10^{-6} / °C$, $E_{al} = 68.9$ GPa , $(\sigma_Y)_{al} = 255$ MPa)

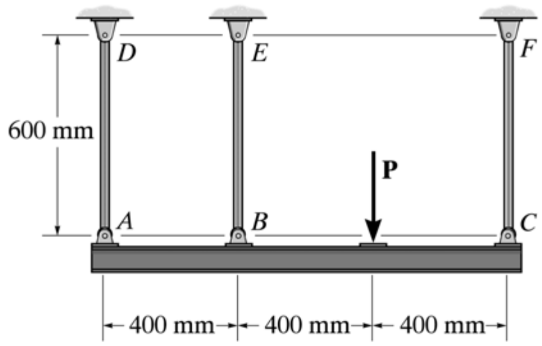


5. 剛性樑由三根直徑 25 mm 的 A-36 鋼桿支撐。若樑支承一 $P = 230$ kN 的負載，

(1) 試求每一鋼桿所受的力。 (10%)

(2) 若負載 $P = 230$ kN 釋放時，試求每一鋼桿的殘留應力。 (10%)

(鋼桿視為完全彈−塑性材料) (A-36 鋼降伏應力 $\sigma_Y = 250$ MPa)



參考解答:

1. $\varepsilon_{\text{lat}}=\dfrac{37.5033-37.5}{37.5}=0.0000880$

$$\sigma=\frac{P}{A}=\frac{-40\cdot 10^{3}}{50\cdot 37.5\cdot 10^{-6}}=-21.33\cdot 10^{6}\ \text{(Pa)}=-21.33\ \text{(MPa)}$$

$$\sigma=E\cdot\varepsilon_{\text{long}}\quad\Rightarrow\varepsilon_{\text{long}}=\frac{\sigma}{E}=\frac{-21.33\cdot 10^{6}}{70\cdot 10^{9}}=-0.00030476$$

$$\nu=-\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}}=-\frac{0.0000880}{-0.00030476}=0.289$$

$$h'=h+\varepsilon_{\text{lat}}\cdot h=50+50\cdot 0.0000880=50.0044\ \text{(mm)}$$

2. (1) 中空棒截面積 $A=50^{2}-40^{2}=900\ (\text{mm}^{2})$

$$\sigma=\frac{P}{A}=\frac{100\cdot 10^{3}}{900\cdot 10^{-6}}=111.11\cdot 10^{6}\ \text{(Pa)}=111.11\ \text{(MPa)}$$ (在彈性範圍內)

由應力-應變圖可知

$$E=\frac{250}{0.00125}=200\cdot 10^{3}\ \text{(MPa)}=200\ \text{(GPa)}$$

$$\delta=\frac{\sigma\cdot L}{E}=\frac{111.11\cdot 10^{-3}\cdot 0.6}{200}=0.33333\cdot 10^{-3}\ \text{(m)}=0.33333\ \text{(mm)}$$

(2) $\sigma=\dfrac{P}{A}=\dfrac{360\cdot 10^{3}}{900\cdot 10^{-6}}=400\cdot 10^{6}\ \text{(Pa)}=400\ \text{(MPa)}$ (已降伏)

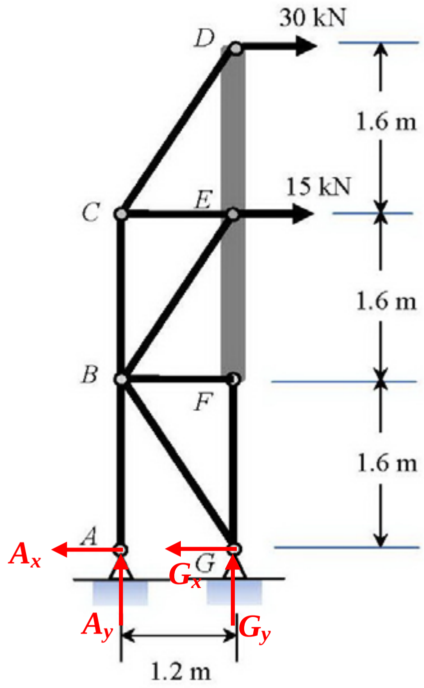
$$\frac{\varepsilon-0.00125}{400-250}=\frac{0.05-0.00125}{500-250}\quad\Rightarrow\varepsilon=0.0305$$

外力釋放後，其沿彈性路徑的恢復量 $\varepsilon_r=\dfrac{400\cdot 10^{-3}}{200}=0.002$

永久應變 $\varepsilon_P=\varepsilon-\varepsilon_r=0.0285$

永久伸長量 $\delta_P=\varepsilon_P\cdot L=0.0285\cdot 600=17.1\ \text{(mm)}$

3.



(1) $\because$ AB 桿件屬二力構件

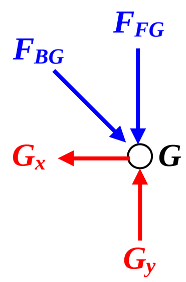
$\therefore \quad A_x = 0$

$\sum F_x = 0 \quad \Rightarrow G_x = 45 \ \text{(kN)}$

$\sum M_A = 0 \quad \Rightarrow G_y \cdot 1.2 - 30 \cdot 4.8 - 15 \cdot 3.2 = 0$

$\Rightarrow G_y = 160 \ \text{(kN)}$

$\sum F_y = 0 \quad \Rightarrow A_y = -160 \ \text{(kN)}$



(2) $\sum F_x = 0 \quad \Rightarrow F_{BG} \cdot \frac{3}{5} - G_x = 0 \quad \Rightarrow F_{BG} = \frac{5}{3} G_x = 75 \ \text{(kN)}$

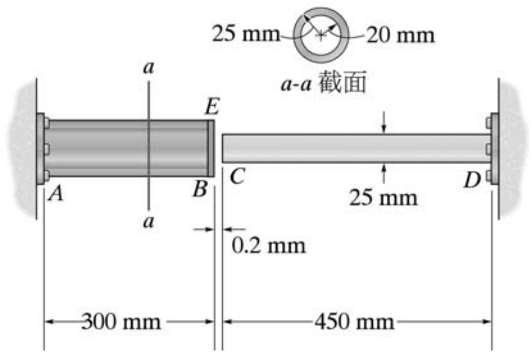
$\sum F_y = 0 \quad \Rightarrow F_{FG} + \frac{4}{5} F_{BG} - G_y = 0 \quad \Rightarrow F_{FG} = G_y - \frac{4}{5} F_{BG} = 100 \ \text{(kN)}$

$\sigma_{allow} = \frac{F_{BG}}{A} \quad \Rightarrow A = \frac{F_{BG}}{\sigma_{allow}} = \frac{75 \cdot 10^3}{120} = 625 \ (\text{mm}^2)$

(3) $G = \frac{E}{2(1+\nu)} \quad \Rightarrow E = 2(1+\nu)G = 2 \cdot (1+0.3) \cdot 77.2 = 200.72 \ \text{(GPa)}$

$\delta_{FG} = \frac{F_{FG} L_{FG}}{AE} = \frac{100 \cdot 10^3 \cdot 1600}{\pi \cdot 20^2 \cdot 10^{-6} \cdot 200.72 \cdot 10^9} = 0.6343 \ \text{(mm)}$

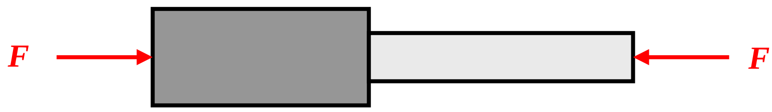
4. (1)



$$(\delta_T)_{AB} = \alpha_{ma} \cdot \Delta T \cdot L_{AB} = 26 \cdot 10^{-6} \cdot (80-30) \cdot 300 = 0.39 \ \text{(mm)}$$

$$(\delta_T)_{CD} = \alpha_{al} \cdot \Delta T \cdot L_{CD} = 24 \cdot 10^{-6} \cdot (80-30) \cdot 450 = 0.54 \ \text{(mm)}$$

∴ 可知當溫度上升至80°C 時，B 點會頂到 C 點



$$\delta = [(\delta_T)_{AB} - (\delta_F)_{AB}] + [(\delta_T)_{CD} - (\delta_F)_{CD}] = 0.2$$

$$\Rightarrow (0.39 - \frac{F \cdot 300}{\pi(25^2 - 20^2) \cdot 10^{-6} \cdot 44.7 \cdot 10^9}) + (0.54 - \frac{F \cdot 450}{\pi \cdot 12.5^2 \cdot 10^{-6} \cdot 68.9 \cdot 10^9}) = 0.2$$

$$\Rightarrow F = 32017.60 \ \text{(N)} = 32.02 \ \text{(kN)}$$

$$\sigma_{AB} = \frac{F}{A_{AB}} = \frac{32017.6}{\pi(25^2 - 20^2) \cdot 10^{-6}} = 45.30 \cdot 10^6 \ \text{(Pa)} = 45.30 \ \text{(MPa)}$$

$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{32017.6}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 65.23 \cdot 10^6 \ \text{(Pa)} = 65.23 \ \text{(MPa)}$$

(2)

$$(\sigma_Y)_{ma} = \frac{F}{A_{AB}} = 152 \text{ MPa} \quad \Rightarrow F = 152 \cdot 10^6 \cdot \pi(25^2 - 20^2) \cdot 10^{-6} = 107442.47 \ \text{(N)}$$

$$(\sigma_Y)_{al} = \frac{F}{A_{CD}} = 255 \text{ MPa} \quad \Rightarrow F = 255 \cdot 10^6 \cdot \pi \cdot 12.5^2 \cdot 10^{-6} = 125172.83 \ \text{(N)}$$

∴ F 最大為 107442.47 (N)

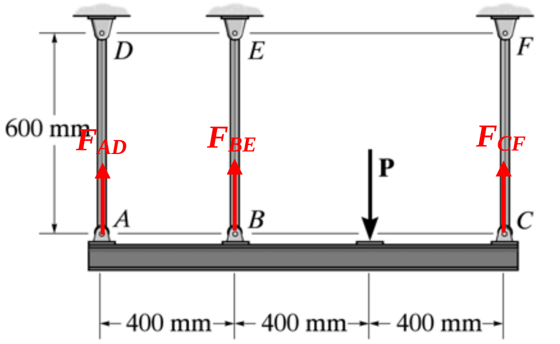
又 $\delta = [(\delta_T)_{AB} - (\delta_F)_{AB}] + [(\delta_T)_{CD} - (\delta_F)_{CD}] = 0.2$

$$\Rightarrow 26 \cdot 10^{-6} \cdot (T-30) \cdot 300 - \frac{107442.47 \cdot 300}{\pi(25^2 - 20^2) \cdot 10^{-6} \cdot 44.7 \cdot 10^9}$$

$$+ 24 \cdot 10^{-6} \cdot (T-30) \cdot 450 - \frac{107442.47 \cdot 450}{\pi \cdot 12.5^2 \cdot 10^{-6} \cdot 68.9 \cdot 10^9} = 0.2$$

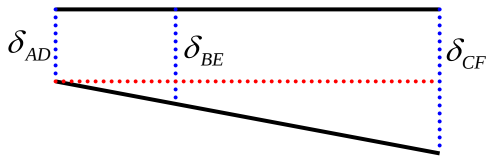
$$\Rightarrow T = 172 \ \text{(°C)}$$

5. (1)



$$\sum F_y = 0 \quad \Rightarrow F_{AD} + F_{BE} + F_{CF} = P \quad \ldots\ldots \text{(a)}$$

$$\sum M_A = 0 \quad \Rightarrow F_{BE} \cdot 400 + F_{CF} \cdot 1200 - P \cdot 800 = 0 \quad \Rightarrow F_{BE} + 3F_{CF} = 2P \ \ldots\ldots \text{(b)}$$



由幾何位移可看出相容方程為 $\dfrac{\delta_{BE} - \delta_{AD}}{400} = \dfrac{\delta_{CF} - \delta_{AD}}{1200}$

$$\Rightarrow 3\delta_{BE} - 2\delta_{AD} - \delta_{CF} = 0$$

$$\Rightarrow 3\frac{F_{BE} \cdot L}{AE} - 2\frac{F_{AD} \cdot L}{AE} - \frac{F_{CF} \cdot L}{AE} = 0$$

$$\Rightarrow 3F_{BE} - 2F_{AD} - F_{CF} = 0 \ \ldots\ldots \text{(c)}$$

由解(a)、(b)、(c)聯立方程可得 $F_{BE} = \dfrac{2}{7}P$，$F_{CF} = \dfrac{4}{7}P$，$F_{AD} = \dfrac{1}{7}P$

$$\therefore \quad F_{BE} = \frac{2}{7} \cdot 230 = 65.71 \ \text{(kN)}$$

$$F_{CF} = \frac{4}{7} \cdot 230 = 131.43 \ \text{(kN)}$$

$$F_{AD} = \frac{1}{7} \cdot 230 = 32.86 \ \text{(kN)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{65.71 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 133.87 \cdot 10^6 \ \text{(Pa)} = 133.87 \ \text{(MPa)} \ < \ \sigma_Y$$

$$\sigma_{CF} = \frac{F_{CF}}{A} = \frac{131.43 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 267.74 \cdot 10^6 \ \text{(Pa)} = 267.74 \ \text{(MPa)} \ > \ \sigma_Y$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{32.86 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 66.94 \cdot 10^6 \ \text{(Pa)} = 66.94 \ \text{(MPa)} \ < \ \sigma_Y$$

$\because \ \sigma_{CF} > \sigma_Y$ 表示 *CF* 桿已降服，故須力量重分配

$$\therefore \quad F'_{CF} = \sigma_Y \cdot A = 250 \cdot 10^6 \cdot \pi \cdot 12.5^2 \cdot 10^{-6} = 122718.46 \ \text{(N)} = 122.72 \ \text{(kN)}$$

$$\sum M_A = 0 \quad \Rightarrow F'_{BE} + 3F'_{CF} = 2P \quad \Rightarrow F'_{BE} = 2P - 3F'_{CF} = 91.84 \ \text{(kN)}$$

$$\sum F_y = 0 \quad \Rightarrow F'_{AD} + F'_{BE} + F'_{CF} = P \quad \Rightarrow F'_{AD} = P - F'_{CF} - F'_{BE} = 15.44 \ \text{(kN)}$$

$$\sigma'_{BE} = \frac{F'_{BE}}{A} = \frac{91.84 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 187.10 \cdot 10^6 \ \text{(Pa)} = 187.10 \ \text{(MPa)} \ < \ \sigma_Y$$

$$\sigma'_{AD} = \frac{F'_{AD}}{A} = \frac{15.44 \cdot 10^3}{\pi \cdot 12.5^2 \cdot 10^{-6}} = 31.45 \cdot 10^6 \ \text{(Pa)} = 31.45 \ \text{(MPa)} \ < \ \sigma_Y$$

(2) $(\sigma_{CF})_r = \sigma'_{CF} - \sigma_{BE} = 250 - 267.74 = -17.74 \ \text{(MPa)}$

$(\sigma_{BE})_r = \sigma'_{BE} - \sigma_{BE} = 187.10 - 133.87 = 53.23 \ \text{(MPa)}$

$(\sigma_{AD})_r = \sigma'_{AD} - \sigma_{AD} = 31.45 - 66.94 = -35.49 \ \text{(MPa)}$