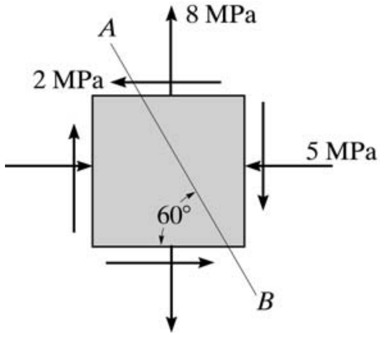
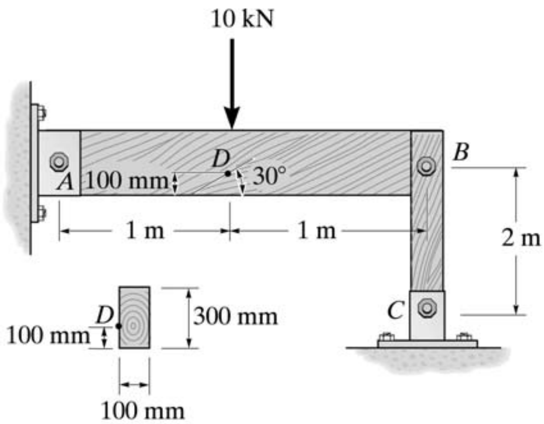
# 國立臺灣海洋大學河海工程研究所2017材料力學2AB班第三次大考參考解答

日期：2017 年 06 月 22 日　　姓名：______________　　學號：______________

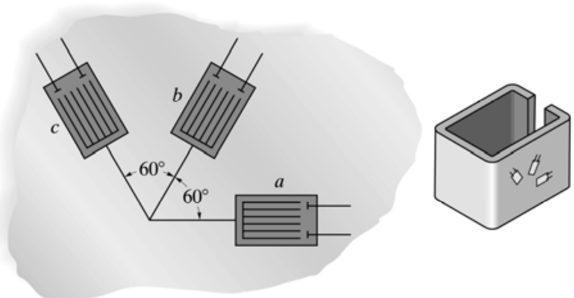
1. 構件中某點位置之應力狀態如圖所示，試畫其莫爾圓圖並求其主應力、最大平面剪應力與作用在 *AB* 斜面的應力分量。(20%)



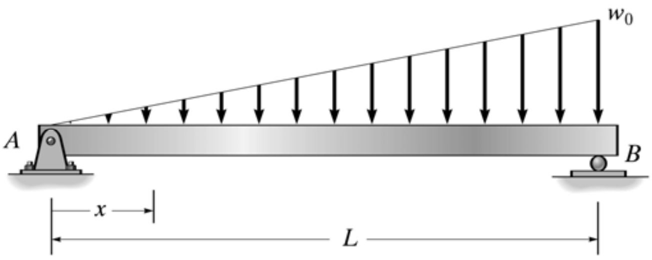

2. (1) 試求 *D* 點處分別垂直及平行作用在纖維上的正應力和剪應力。此處纖維如圖與水平呈30°。*D* 點恰位於外力 10 kN 的左側。(12%)
   (2) 試求 *D* 點處的主應力。(8%)



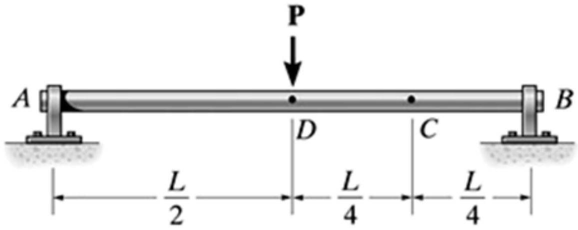

3. 一60°應變菊花座貼附在托臂架上。各量規上的讀數分別為$\varepsilon_a = -100\times10^{-6}$，$\varepsilon_b = 250\times10^{-6}$和$\varepsilon_c = 150\times10^{-6}$。試求: (a) 主應變 (10%) (b) 最大同平面剪應變及其相關的平均正應變。(10%)



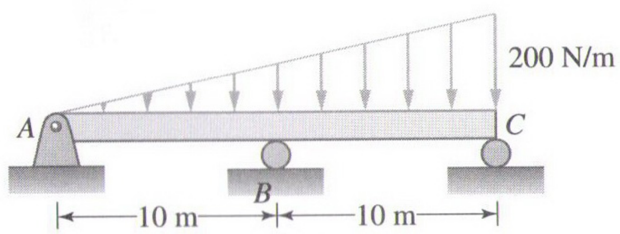
4. (1) 試求下圖簡支樑的彈性曲線方程式並計算 $\theta_A$ 與 $\theta_B$ 為何? (12%)
   (2) 試求最大撓曲($v_{\max}$)發生在何處，即 $x=$？ 並計算 $v_{\max}$ 為何? (8%)




5. 試以力矩-面積法計算
   (1) $\theta_A$ 與 $v_{\max}$。 (10%)
   (2) $\theta_C$ 與 $v_C$。 (10%)




6. 試求下圖所有支承反力之大小。(10%)




7. (1) 上完了一學期的材料力學，對於這門課在學習上有何心得或感想? (5%)
   (2) 對於老師的教學方式或是要如何協助同學們學習好這門課有何建議? (5%)
   (有寫才有分)

**(參考公式)**

**平面應力轉換方程式**

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

**平面應變轉換方程式**

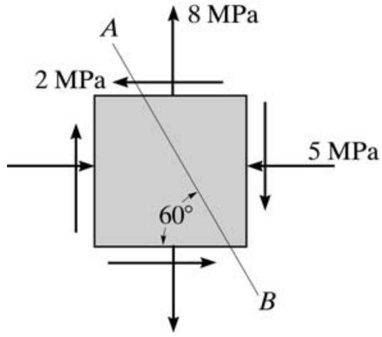
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

**彎曲公式：** $\sigma = -\dfrac{My}{I}$， **剪力公式：** $\tau = \dfrac{VQ}{It}$

參考解答:

**1. 構件中某點位置之應力狀態如圖所示，試畫其莫爾圓圖並求其主應力、最大平面剪應力與作用在 *AB* 斜面的應力分量。(20%)**



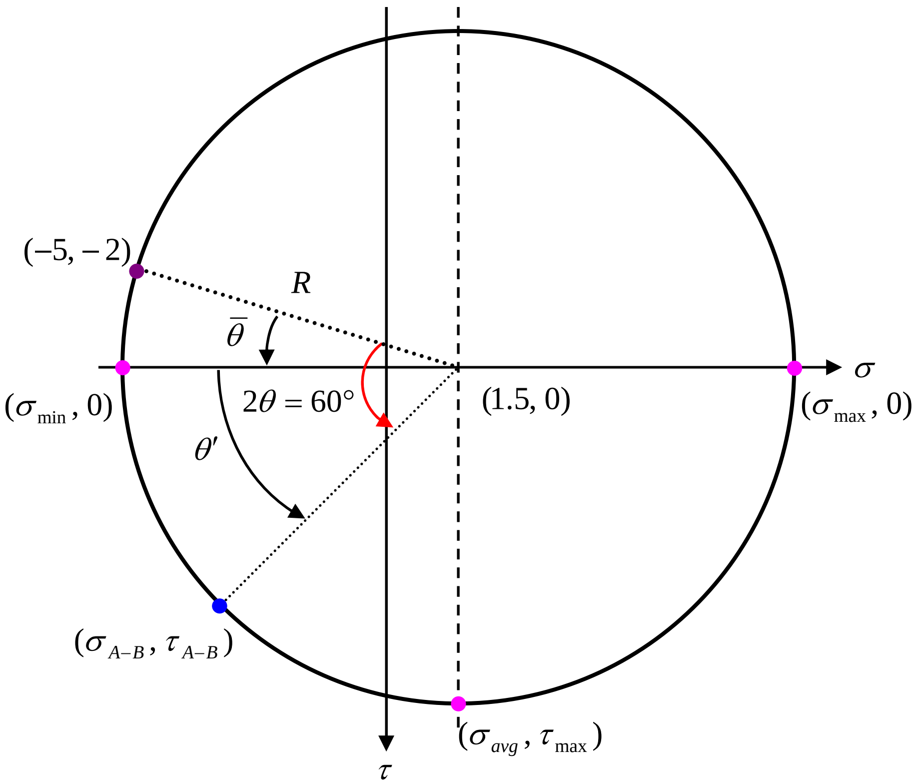

由圖可看出，所要求解的 $A-B$ 面為元素逆時針旋轉 $30°$ 後所得的平面

$\therefore$ $\sigma_x = -5$ MPa，$\sigma_y = 8$ MPa，$\tau_{xy} = -2$ MPa 且 $\theta = 30°$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 1.5\ (\text{MPa})$$

圓心位置為 $(\sigma_{avg}, 0) = (1.5, 0)$

所以莫爾圓為




$\therefore$ 半徑 $R = \sqrt{(-5-1.5)^2 + (-2)^2} = 6.80$

主應力 $\sigma_{max} = \sigma_{avg} + R = 8.3\ (\text{MPa})$

$$\sigma_{\min} = \sigma_{avg} - R = -5.3\ \text{(MPa)}$$

最大剪應力 $\tau_{\max} = R = 6.8\ \text{(MPa)}$

$$\bar{\theta} = \tan^{-1}\frac{2}{6.5} = 0.2985\ \text{(rad)} = 17.10°$$

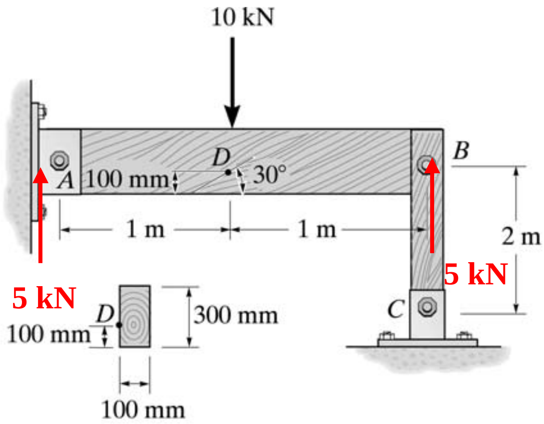
$$\theta' = 60° - 17.10° = 42.90°$$

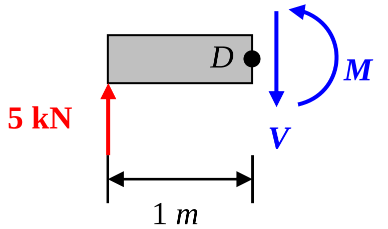
$$\therefore\ \sigma_{A-B} = 1.5 - 6.8 \cdot \cos 42.9° = -3.48\ \text{(MPa)}$$

$$\tau_{A-B} = 6.8 \sin 42.9° = 4.63\ \text{(MPa)}$$

**2. (1) 試求 *D* 點處分別垂直及平行作用在纖維上的正應力和剪應力。此處纖維如圖與水平呈 30°。*D* 點恰位於外力 10 kN 的左側。(12%)**

**(2) 試求 *D* 點處的主應力。(8%)**



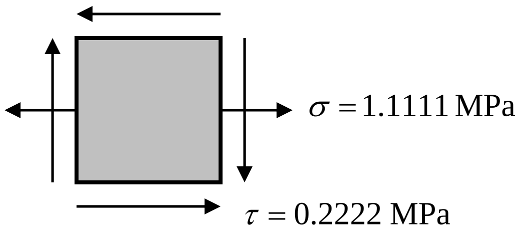

(1) 由圖可知 $V = 5\ \text{(kN)}$ 與 $M = 5\ \text{(kN} \cdot \text{m)}$

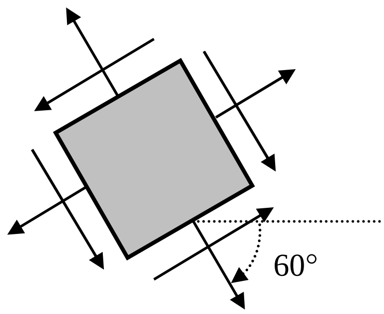
$$I = \frac{1}{12} 0.1 \cdot 0.3^3 = 225 \cdot 10^{-6}\ (\text{m}^4)$$

$$Q = A \cdot \bar{y} = (0.1 \cdot 0.1) \cdot 0.1 = 0.001\ (\text{m}^3)$$

$$\tau = \frac{VQ}{It} = \frac{5 \cdot 10^3 \cdot 0.001}{225 \cdot 10^{-6} \cdot 0.1} = 0.2222 \cdot 10^6\ \text{(Pa)} = 0.2222\ \text{(MPa)}$$

$$\sigma = -\frac{My}{I} = -\frac{5 \cdot 10^3 \cdot (-0.05)}{225 \cdot 10^{-6}} = 1.1111 \cdot 10^6\ \text{(Pa)} = 1.1111\ \text{(MPa)}$$




$$\therefore\ \sigma_x = 1.1111\ \text{(MPa)},\ \sigma_y = 0,\ \tau_{xy} = -0.2222\ \text{(MPa)}$$

且 $\theta$ 為順時鐘方向轉 60° 即 $\theta = -60°$

$$\sigma_{x'} = \frac{1.1111+0}{2} + \frac{1.1111-0}{2}\cos(-120°) - 0.2222\cdot\sin(-120°) = 0.4702\ \text{(MPa)}$$

$$\tau_{x'y'} = -\frac{1.1111-0}{2}\sin(-120°) - 0.2222\cos(-120°) = 0.5922\ \text{(MPa)}$$

(2) $\tan 2\theta_p = \dfrac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \dfrac{-0.2222}{\frac{1.1111-0}{2}} = -0.4$

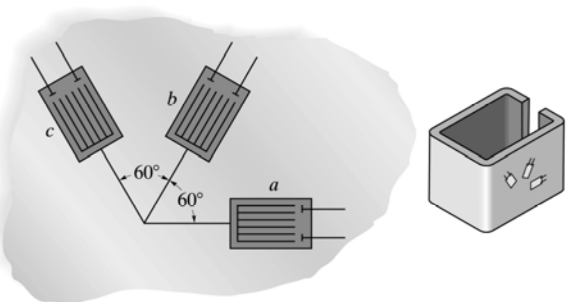
$\Rightarrow 2\theta_p = \tan^{-1}(-0.4) = -0.3805 = -21.8014°$

$\Rightarrow \theta_p = \tan^{-1}(-0.4) = -0.3805 = -10.9°$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \frac{1.1111+0}{2} \pm \sqrt{(\frac{1.1111-0}{2})^2 + 0.2222^2}$$

$\therefore\ \sigma_1 = 1.1539\ \text{(MPa)}$，$\sigma_2 = -0.0428\ \text{(MPa)}$

**3.** **一**$60°$**應變菊花座貼附在托臂架上。各量規上的讀數分別為**$\varepsilon_a = -100\times10^{-6}$，$\varepsilon_b = 250\times10^{-6}$**和**$\varepsilon_c = 150\times10^{-6}$**。試求: (a) 主應變 (10%) (b) 最大同平面剪應變及其相關的平均正應變。(10%)**




$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_a = -100\times10^{-6} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 0° + \frac{\gamma_{xy}}{2}\sin 0° \quad \Rightarrow \xi_x = -100\times10^{-6}$$

$$\varepsilon_b = 250\times10^{-6} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 120° + \frac{\gamma_{xy}}{2}\sin 120°$$

$$\Rightarrow \frac{3}{2}\varepsilon_y + \frac{\sqrt{3}}{2}\gamma_{xy} = 550\times10^{-6}\ \ldots\ldots\ldots(1)$$

$$\varepsilon_c = 150\times10^{-6} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 240° + \frac{\gamma_{xy}}{2}\sin 240°$$

$$\Rightarrow \frac{3}{2}\varepsilon_y - \frac{\sqrt{3}}{2}\gamma_{xy} = 350\times10^{-6}\ \ldots\ldots\ldots(2)$$

由 $(1)+(2)$ 可得 $\varepsilon_y = 300\times10^{-6}$

由 $(1)-(2)$ 可得 $\gamma_{xy} = \dfrac{200\sqrt{3}}{3}\times10^{-6} = 115.47\times10^{-6}$

(1) $\varepsilon_{1,2} = \dfrac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{(\dfrac{\varepsilon_x-\varepsilon_y}{2})^2 + (\dfrac{\gamma_{xy}}{2})^2} = 1700\times10^{-6}$ and $-300\times10^{-6}$

$$\Rightarrow \varepsilon_{1,2} = \frac{-100+300}{2} \pm \sqrt{(\frac{-100-300}{2})^2 + (\frac{115.47}{2})^2}\times10^{-6}$$

$$\Rightarrow \varepsilon_1 = 308.17\times10^{-6} \quad \text{and} \quad \varepsilon_2 = -108.17\times10^{-6}$$

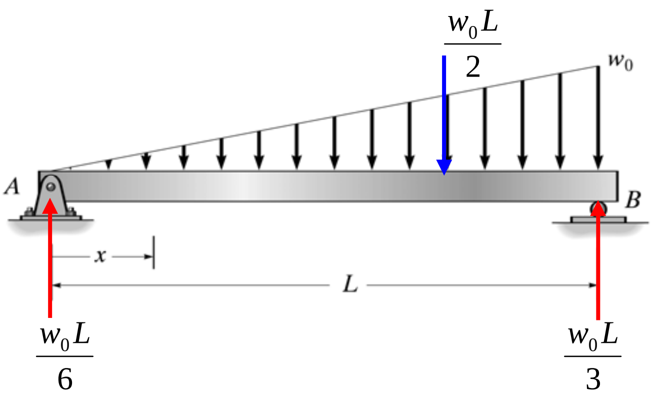
(2) $\dfrac{\gamma_{\max}}{2} = \sqrt{(\dfrac{\varepsilon_x-\varepsilon_y}{2})^2 + (\dfrac{\gamma_{xy}}{2})^2} = 208.17\times10^{-6}$

$$\Rightarrow \gamma_{\max} = 416.34\times10^{-6}$$

(3) $\varepsilon_{avg} = \dfrac{\varepsilon_1+\varepsilon_2}{2} = \dfrac{-100+300}{2}\times10^{-6} = 100\times10^{-6}$

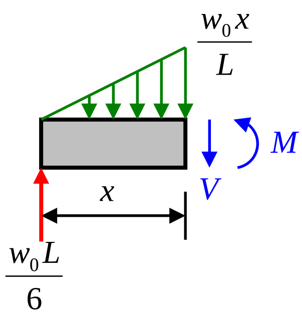
**4. (1) 試求下圖簡支樑的彈性曲線方程式並計算 $\theta_A$ 與 $\theta_B$ 為何? (12%)**

**(2) 試求最大撓曲($v_{\max}$)發生在何處，即 $x=$? 並計算 $v_{\max}$ 為何? (8%)**




(1) 由彎矩平衡可得

$$M = \frac{w_0L}{6}x - \frac{w_0x^3}{6L}$$




又 $v'' = \dfrac{M}{EI} = \dfrac{w_0L}{6EI}x - \dfrac{w_0x^3}{6LEI}$

$$v' = \frac{w_0L}{12EI}x^2 - \frac{w_0}{24LEI}x^4 + C_1$$

$$v = \frac{w_0L}{36EI}x^3 - \frac{w_0}{120LEI}x^5 + C_1x + C_2$$

由邊界條件可知:

$x=0$，$v(0)=0 \quad \Rightarrow C_2 = 0$

$$x = L\text{，}\ v(L) = 0 \quad \Rightarrow C_1 = -\frac{7w_0L^3}{360EI}$$

$$\therefore v(x) = \frac{w_0L}{36EI}x^3 - \frac{w_0}{120LEI}x^5 - \frac{7w_0L^3}{360EI}x$$

$$v'(x) = \frac{w_0L}{12EI}x^2 - \frac{w_0}{24LEI}x^4 - \frac{7w_0L^3}{360EI}$$

$$\theta_A = v'(0) = -\frac{7w_0L^3}{360EI}$$

$$\theta_B = v'(L) = \frac{w_0L^3}{45EI}$$

(2) 最大撓曲發生在 $v' = 0$ 之處

$$\therefore v' = \frac{w_0L}{12EI}x^2 - \frac{w_0}{24LEI}x^4 - \frac{7w_0L^3}{360EI} = 0$$

$$\Rightarrow 15x^4 - 30L^2x^2 + 7L^4 = 0$$

$$\Rightarrow 15(x^2 - L^2)^2 - 8L^4 = 0$$

$$\Rightarrow x^2 = L^2 \pm \sqrt{\frac{8}{15}}L^2 = 1.7303L^2 \text{ or } 0.2697L^2$$

$$\Rightarrow x^2 = L^2 \pm \sqrt{\frac{8}{15}}L^2 = 1.7303L^2 \text{ or } 0.2697L^2$$

$$\Rightarrow x = 1.3154L\ (\text{不合}) \quad \text{or} \quad x = 0.5193L$$

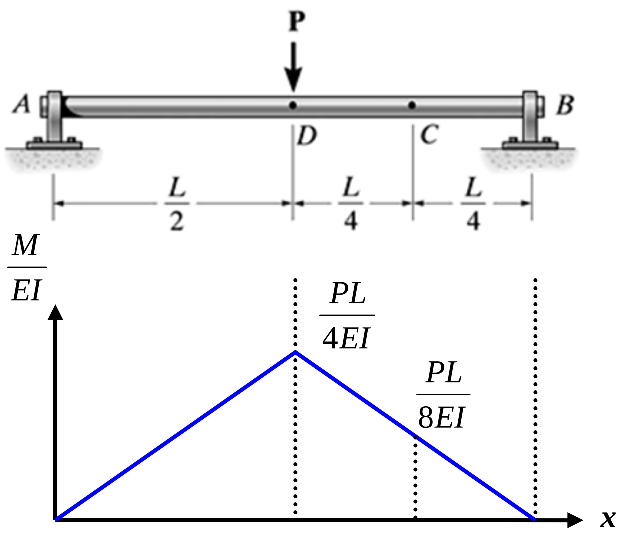
$\therefore$ 最大撓曲發生在 $x = 0.5193L$ 處

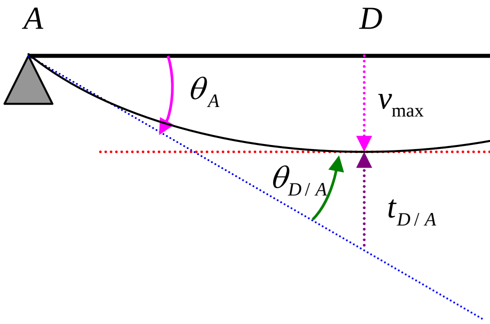
$$v_{max} = v(0.5193L) = -0.00652\frac{w_0L^4}{EI}$$

**5. 試以力矩-面積法計算**

**(1)** $\theta_A$ 與 $v_{max}$ 。 **(10%)**

**(2)** $\theta_C$ 與 $v_C$ 。 **(10%)**






(1) $|\theta_A| = |\theta_{D/A}| = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{PL}{4EI} = \frac{PL^2}{16EI}$

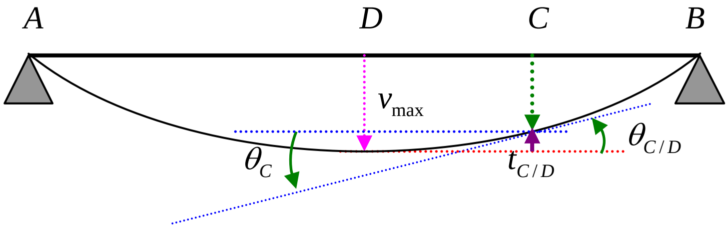
又 $\theta_A$ 為順時鐘方向旋轉

$\therefore\ \theta_A = -\frac{PL^2}{16EI}$

$|v_{\max}| = |\theta_A|\cdot\frac{L}{2} - |t_{D/A}| = \frac{PL^2}{16EI}\cdot\frac{L}{2} - \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{PL}{4EI}\cdot\frac{L}{6} = \frac{PL^3}{48EI}$

又在 $D$ 點位移往下

$\therefore\ v_{\max} = -\frac{PL^3}{48EI}$



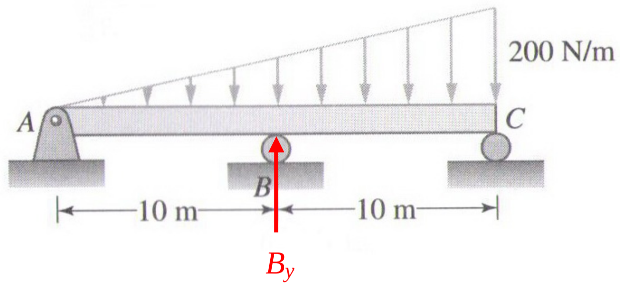

(2) $\theta_{C/D} = \frac{1}{2}\cdot\left(\frac{PL}{4EI}+\frac{PL}{8EI}\right)\frac{L}{4} = \frac{3PL^2}{64EI}$

$\theta_C = \theta_{C/D} = \frac{3PL^2}{64EI}$

$t_{C/D} = \left(\frac{PL}{8EI}\cdot\frac{L}{4}\right)\cdot\frac{L}{8} + \left(\frac{1}{2}\cdot\frac{PL}{8EI}\cdot\frac{L}{4}\right)\cdot\frac{L}{6} = \frac{5PL^3}{768EI}$

$v_C = v_{\max} + t_{C/D} = -\frac{PL^3}{48EI} + \frac{5PL^3}{768EI} = -\frac{11PL^3}{768EI}$

**6. 試求下圖所有支承反力之大小。(10%)**

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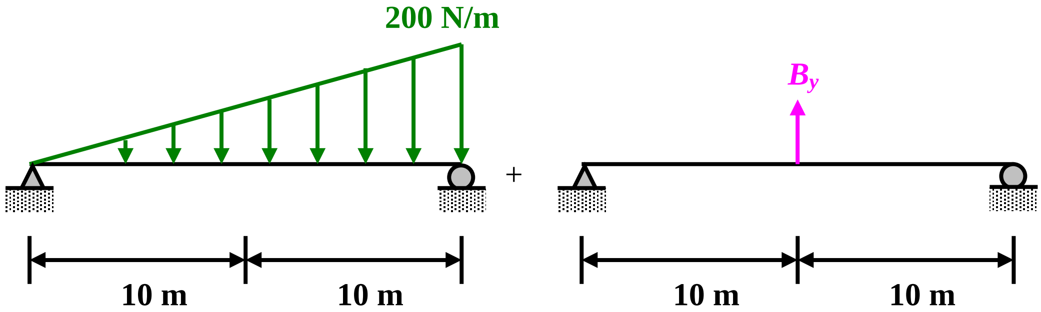


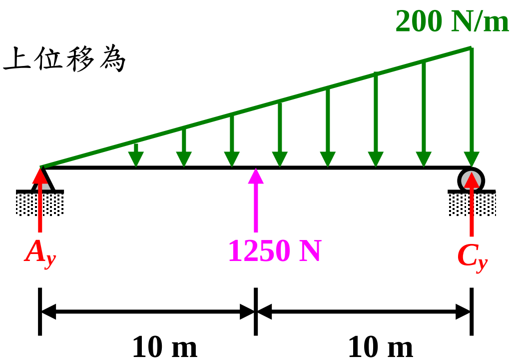

圖 A　　　　圖 B

由第 4 題結果可知圖 A 在 $\frac{L}{2}$ 處(B 點)向下位移為

$$v(\frac{L}{2}) = \frac{w_0 L^4}{8 \cdot 36EI} - \frac{w_0 L^4}{32 \cdot 120EI} - \frac{7w_0 L^4}{2 \cdot 360EI} = -\frac{5w_0 L^4}{768EI}$$

$$\therefore\ v(10) = -\frac{5 \cdot 200 \cdot 20^4}{768EI} = -\frac{625000}{3EI}$$

由第 5 題結果可知圖 B 在 $\frac{L}{2}$ 處(B 點)向上位移為




$$v(\frac{L}{2}) = v_{\max} = \frac{PL^3}{48EI}$$

$$\therefore\ \ v(10) = \frac{B_y \cdot 20^3}{48EI} = \frac{500B_y}{3EI}$$

又 B 點為滾支承，即 B 點位移為零

$$\therefore\ \ -\frac{625000}{3EI} + \frac{500B_y}{3EI} = 0 \quad \Rightarrow B_y = 1250\ \text{(N)}$$

$$\sum M_A = 0 \quad \Rightarrow C_y \cdot 20 + 1250 \cdot 10 - \frac{1}{2} \cdot 20 \cdot 200 \cdot \frac{40}{3} = 0 \quad \Rightarrow C_y = \frac{2125}{3}\ \text{(N)}$$

$$\sum F_y = 0 \quad \Rightarrow A_y = \frac{1}{2} \cdot 20 \cdot 200 - C_y - 1250 = 0 \quad \Rightarrow A_y = \frac{125}{3}\ \text{(N)}$$