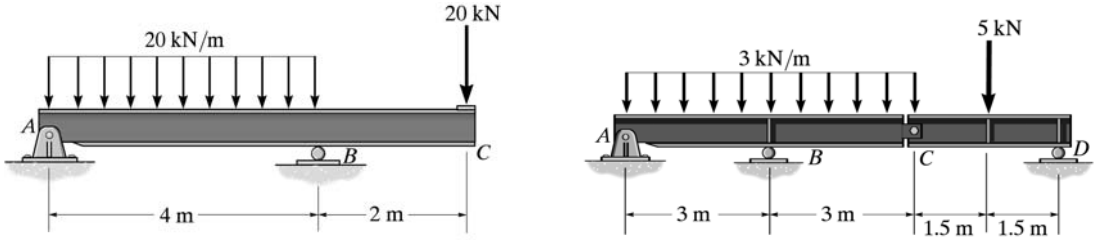
國立臺灣海洋大學河海工程研究所 2017 材料力學 2AB 班第二次大考參考解答

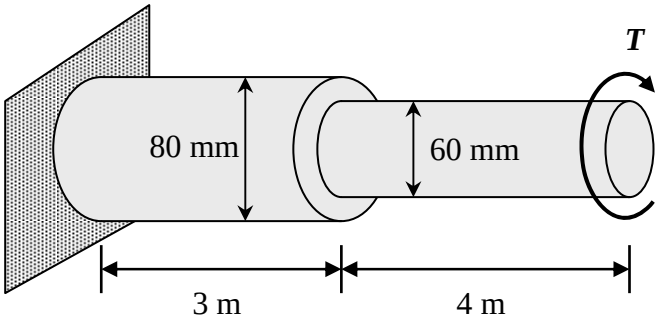
日期：2017 年 05 月 22 日　　姓名：＿＿＿＿＿＿　　學號：＿＿＿＿＿＿

1. 請畫出下面各樑的的剪力圖和彎矩圖。(20%)

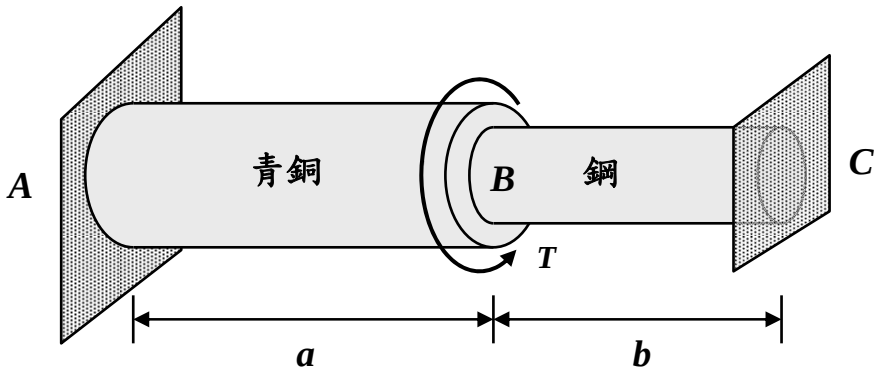


(a)　　(b)

2. 如圖所示之階梯鋼軸承受扭矩 T。若該鋼軸之容許剪應力為 12 MPa 且自由端的扭轉角需小於 4°，試求最大容許的扭矩 T。已知鋼的 $G = 83$ GPa 。 (20%)



3. 如圖所示之複合軸兩端固定於剛性壁。其中青銅段 AB 的直徑為 75 mm 及 $G = 35$ GPa ，而鋼段 BC 的直徑則為 50 mm 及 $G = 83$ GPa 。已知 $a = 2$ m 和 $b = 1.5$ m。若青銅與鋼之最大剪應力分別不超過 60 MPa 和 80 MPa ，試求所能施加之最大扭矩 T 。(20%)

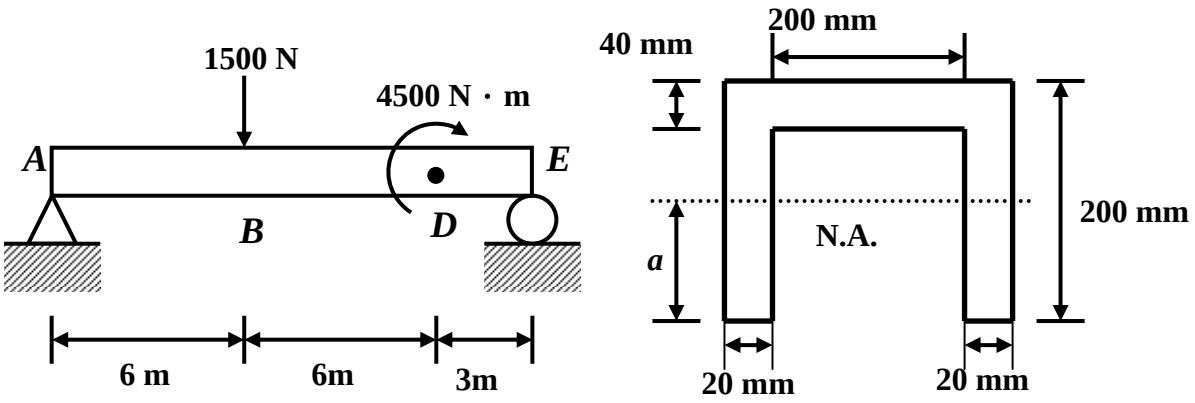


4. 一樑 ABC 及其作用力與樑之斷面如下圖所示，試求:

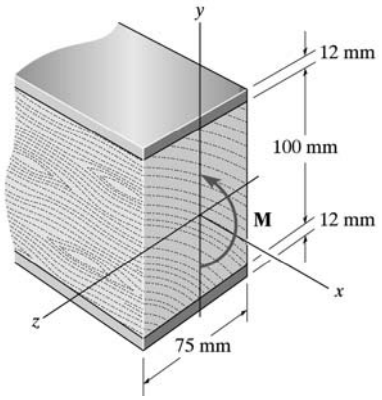
(1) 此樑斷面的中性軸位置 (a = ?) (5%)

(2) 對此斷面中性軸的面積慣性矩 I = ? (5%)

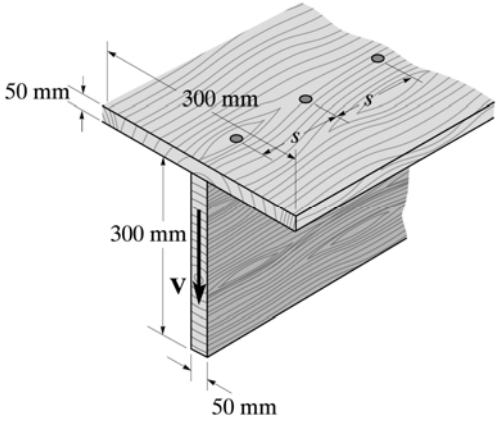
(3) 此樑的最大拉應力和最大壓應力為何? (10%)



5. 一木樑是用 A-36 鋼條在頂部及底部補強，如圖示。若其容許應力分別為 $(\sigma_{\text{allow}})_{\text{st}} = 150\,\text{MPa}$，$(\sigma_{\text{allow}})_{\text{w}} = 14\,\text{MPa}$，試求可支承之彎矩 M。
(A-36 鋼: E_{st} = 200GPa; 木樑: E_w = 9.65GPa) (20%)

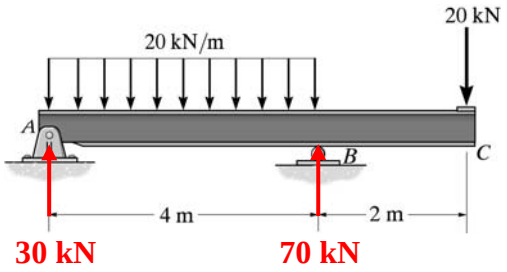


6. T 型樑以釘子連結，如下圖所示。若各釘均可支承 4.5 kN 的剪力，試求樑可支承的最大剪力 V 以及以 5 mm 為間隔尺寸之對應的最大釘距 s。木板的容許剪應力為 $\tau_{\text{allow}} = 3\,\text{MPa}$ 。(20%)

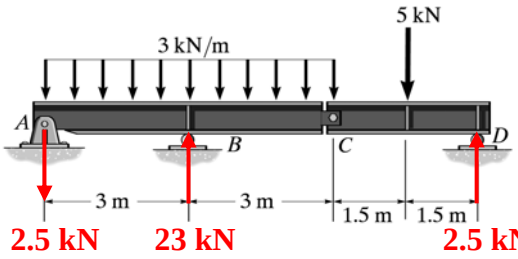


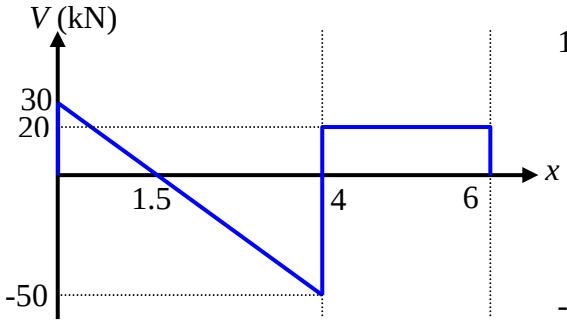
參考解答:

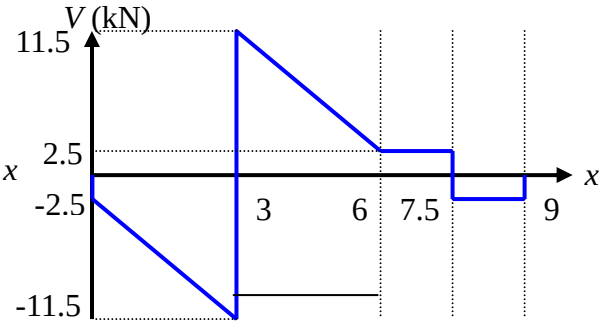
1. (a)

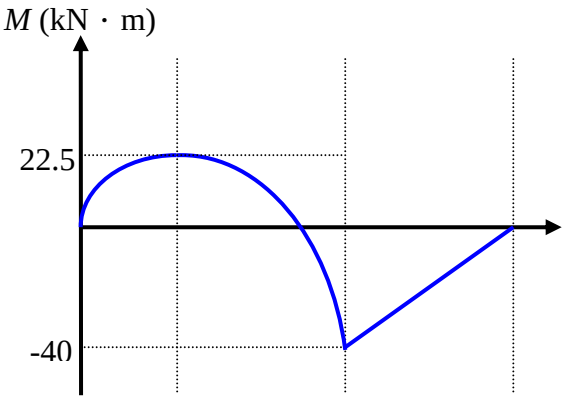


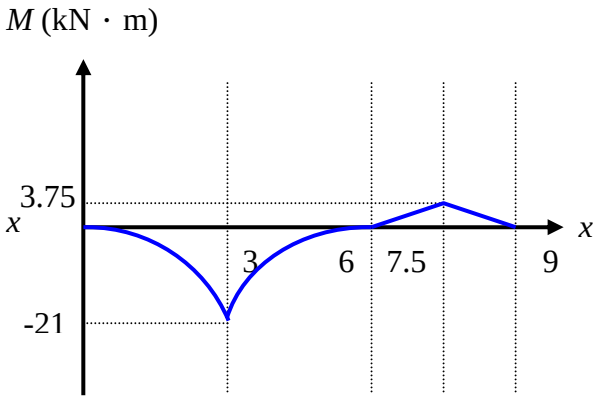
(b)



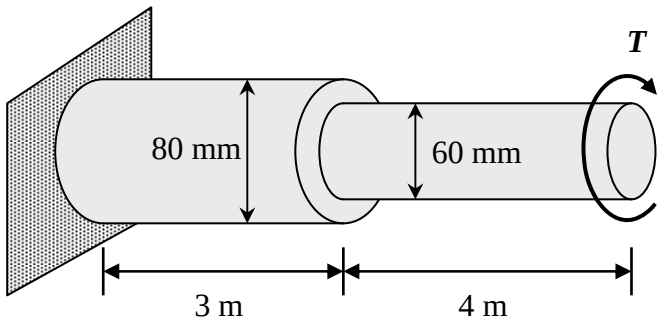








2.



由 $\tau_{max} = \dfrac{Tc}{J}$ 可知 J 較小則 τ 較大

$\therefore \tau_{max}$ 發生在細管段的表面處

$$\tau_{\max}=\frac{Tc}{J}\quad\Rightarrow T=\frac{J\cdot\tau_{\max}}{c}=\frac{(\frac{\pi}{2}\cdot 30^4\cdot 10^{-12})\cdot(12\cdot 10^{-6})}{30\cdot 10^{-3}}=508.94\ \ (\mathrm{N\cdot m})$$

$$\phi=\sum\frac{TL}{JG}\quad\Rightarrow 4\cdot\frac{\pi}{180}=\frac{T\cdot 3}{(\frac{\pi}{2}\cdot 40^4\cdot 10^{-12})\cdot(83\cdot 10^9)}+\frac{T\cdot 4}{(\frac{\pi}{2}\cdot 30^4\cdot 10^{-12})\cdot(83\cdot 10^9)}$$

$$\Rightarrow 4\cdot\frac{\pi}{180}=\frac{T\cdot 3}{106240\pi}+\frac{T\cdot 4}{33615\pi}$$

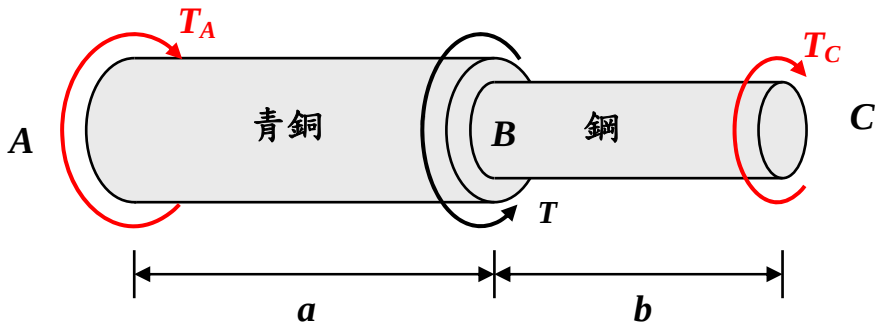
$$\Rightarrow 4\cdot\frac{\pi}{180}=\frac{1267\,T}{8605440\pi}$$

$$\Rightarrow 4\cdot\frac{\pi}{180}=\frac{1267\,T}{8605440\pi}$$

$$\Rightarrow T=1489.65\ \ (\mathrm{N\cdot m})$$

∴最大容許的扭矩 $T=508.94\ \ (\mathrm{N\cdot m})$

3.



$T_A+T_C=T$

相容方程式：$\phi=0$

$$\Rightarrow\phi=\frac{T_A\,a}{J_{AB}G_{青銅}}-\frac{T_C\,b}{J_{BC}G_{鋼}}=0$$

$$\Rightarrow T_C=\frac{a}{b}\frac{J_{BC}}{J_{AB}}\frac{G_{鋼}}{G_{青銅}}T_A=\frac{2}{1.5}\cdot\frac{\frac{\pi}{2}25^4\cdot 10^{-12}}{\frac{\pi}{2}37.5^4\cdot 10^{-12}}\cdot\frac{83\cdot 10^9}{35\cdot 10^9}T_A$$

$$\Rightarrow T_C=0.6246T_A$$

$\therefore\ \ T_A+T_C=T\ \ \Rightarrow T_A=0.6155T\qquad\Rightarrow T_C=0.6246T_A=0.3845T$

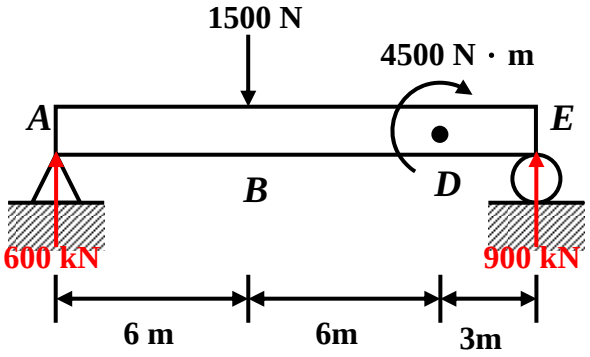
又 $(\tau_{青銅})_{\max}=60\ \mathrm{MPa}$ 與 $(\tau_{鋼})_{\max}=80\ \mathrm{MPa}$

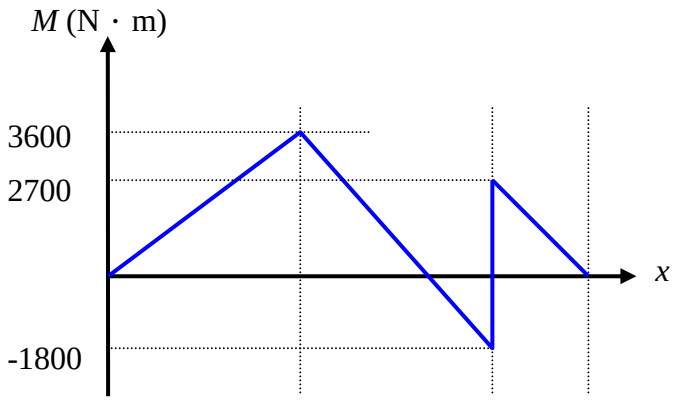
$$\therefore\quad (\tau_{\text{青銅}})_{\max}=\frac{0.6155T\cdot 37.5\cdot 10^{-3}}{\frac{\pi}{2}37.5^4\cdot 10^{-12}}=60\cdot 10^6\quad\Rightarrow T=8074.89\ \ (\text{N}\cdot\text{m})$$

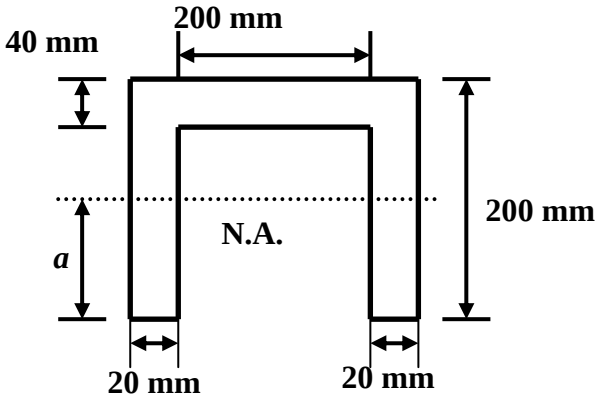
$$(\tau_{\text{鋼}})_{\max}=\frac{0.3845T\cdot 25\cdot 10^{-3}}{\frac{\pi}{2}25^4\cdot 10^{-12}}=80\cdot 10^6\quad\Rightarrow T=5106.62\ \ (\text{N}\cdot\text{m})$$

$\therefore$ 所能施加之最大扭矩 $T=5106.62\ \ (\text{N}\cdot\text{m})$

4.







(1) $\bar{y}=\dfrac{\sum y_i A_i}{\sum A_i}=\dfrac{0.18\cdot(0.2\cdot 0.04)+0.1\cdot(0.02\cdot 0.2)\cdot 2}{(0.2\cdot 0.04)+(0.02\cdot 0.2)\cdot 2}=0.14\ (\text{m})$

$\therefore\quad a=\bar{y}=0.14\ (\text{m})$

(2) $I = \left[\frac{1}{12}\cdot 0.02\cdot 0.2^3 + 0.02\cdot 0.2\cdot(0.14-0.1)^2\right]\cdot 2$

$$+\frac{1}{12}\cdot 0.2\cdot 0.04^3 + 0.2\cdot 0.04\cdot(0.18-0.14)^2$$

$$\Rightarrow I = 53.3333\cdot 10^{-6}\ \ (\mathrm{m}^4)$$

(3) $\sigma = -\frac{My}{I}$

$$(\sigma_B)_{top} = -\frac{3600\cdot 0.06}{53.3333\cdot 10^{-6}} = -4.05\cdot 10^6\ \ (\mathrm{Pa}) = -4.05\ \ (\mathrm{MPa})$$

$$(\sigma_B)_{bottom} = -\frac{3600\cdot(-0.14)}{53.3333\cdot 10^{-6}} = 9.45\cdot 10^6\ \ (\mathrm{Pa}) = 9.45\ \ (\mathrm{MPa})$$

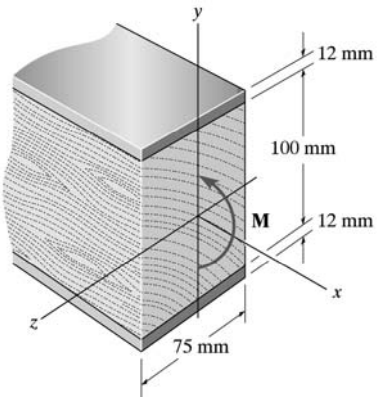
$$(\sigma_{D^-})_{top} = -\frac{(-1800)\cdot 0.06}{53.3333\cdot 10^{-6}} = 2.025\cdot 10^6\ \ (\mathrm{Pa}) = 2.025\ \ (\mathrm{MPa})$$

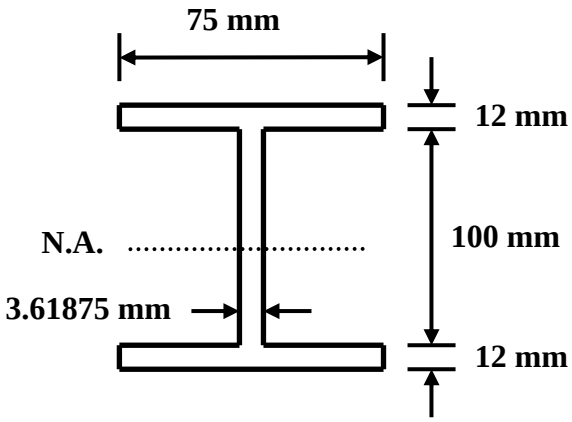
$$(\sigma_{D^-})_{bottom} = -\frac{(-1800)\cdot(-0.14)}{53.3333\cdot 10^{-6}} = -4.725\cdot 10^6\ \ (\mathrm{Pa}) = -4.725\ \ (\mathrm{MPa})$$

此樑的最大拉應力在 B 點底部為 9.45 MPa

最大壓應力在 D 點左邊底部為 4.725 MPa

5.





$$n = \frac{E_w}{E_{st}} = \frac{9.65}{200} = 0.04825$$

$$b_{st} = nb_w = 0.04825\cdot 75 = 3.61875$$

$$I = \frac{1}{12}\cdot 75\cdot 124^3 - \frac{1}{12}\cdot(75-3.61875)\cdot 100^3$$

$$= 5.9680\cdot 10^6\ \ (\mathrm{mm}^4) = 5.9680\cdot 10^{-6}\ \ (\mathrm{m}^4)$$

若鋼條產生破壞則

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{Mc}{I} \qquad \Rightarrow 150\cdot 10^6 = \frac{M\cdot 62\cdot 10^{-3}}{5.9680\cdot 10^{-6}}$$

$$\Rightarrow M = 14.44\cdot 10^3\ (\mathrm{N}\cdot\mathrm{m}) = 14.44\ (\mathrm{kN}\cdot\mathrm{m})$$

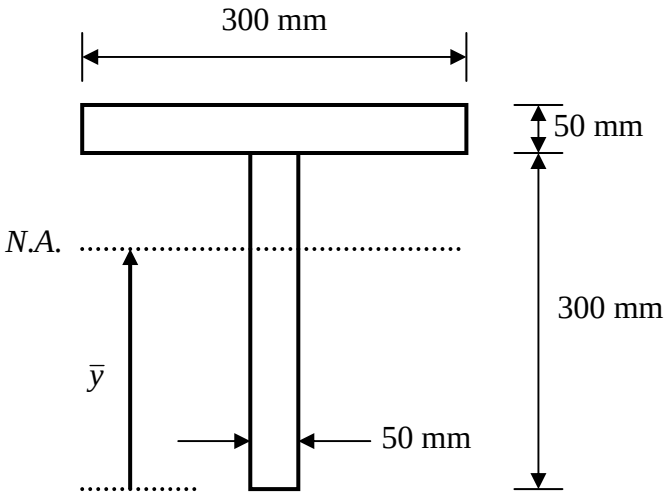
若木樑產生破壞則

$$(\sigma_{\text{allow}})_{\text{w}} = n\frac{My}{I} \quad \Rightarrow 14\cdot 10^{6} = 0.04825\cdot\frac{M\cdot 50\cdot 10^{-3}}{5.9680\cdot 10^{-6}}$$

$$\Rightarrow M = 34.63\cdot 10^{3}\ (\text{N}\cdot\text{m}) = 34.63\ (\text{kN}\cdot\text{m})$$

∴ 此樑可承受彎矩為 $M = 14.44\ (\text{kN}\cdot\text{m})$

6.



$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{325\cdot(300\cdot 50)+150\cdot(50\cdot 300)}{(300\cdot 50)+(50\cdot 300)} = 237.5\ (\text{mm})$$

$$I = \frac{1}{12}\cdot 50\cdot 300^{3} + 50\cdot 300\cdot(237.5-150)^{2}$$

$$+\frac{1}{12}\cdot 300\cdot 50^{3} + 300\cdot 50\cdot(325-237.5)^{2}$$

$$\Rightarrow I = 345.3125\cdot 10^{6}\ \ (\text{mm}^{4}) = 345.3125\cdot 10^{-6}\ \ (\text{m}^{4})$$

最大剪應力出現在中性軸處

$$\therefore\ Q_{\max} = \bar{y}A = 50\cdot 237.5\cdot\frac{237.5}{2}$$

$$= 1.4102\cdot 10^{6}\ (\text{mm}^{3}) = 1.4102\cdot 10^{-3}\ (\text{m}^{3})$$

$$\tau_{\text{allow}} = \frac{VQ_{\max}}{It} \quad \Rightarrow 3\cdot 10^{6} = \frac{V\cdot 1.4102\cdot 10^{-3}}{345.3125\cdot 10^{-6}\cdot 0.05}$$

$$\Rightarrow V = 36.73\cdot 10^{3}\ (\text{N}) = 36.73\ (\text{kN})$$

∴可支承的最大剪力 $V = 36.73\ (\text{kN})$

$$Q = \bar{y}A = 300\cdot 50\cdot(325-237.5)$$

$$= 1.3125\cdot 10^{6}\ (\text{mm}^{3}) = 1.3125\cdot 10^{-3}\ (\text{m}^{3})$$

$$q = \frac{VQ}{I} \quad \Rightarrow \frac{4.5\cdot 10^{3}}{s} = \frac{36.73\cdot 10^{3}\cdot 1.3125\cdot 10^{-3}}{345.3125\cdot 10^{-6}}$$

$$\Rightarrow s = 32.23\cdot 10^{-3}\ (\text{m}) = 32.23\ (\text{mm})$$

以 5 mm 為間隔尺寸之對應的最大釘距 $s = 35\ (\text{mm})$