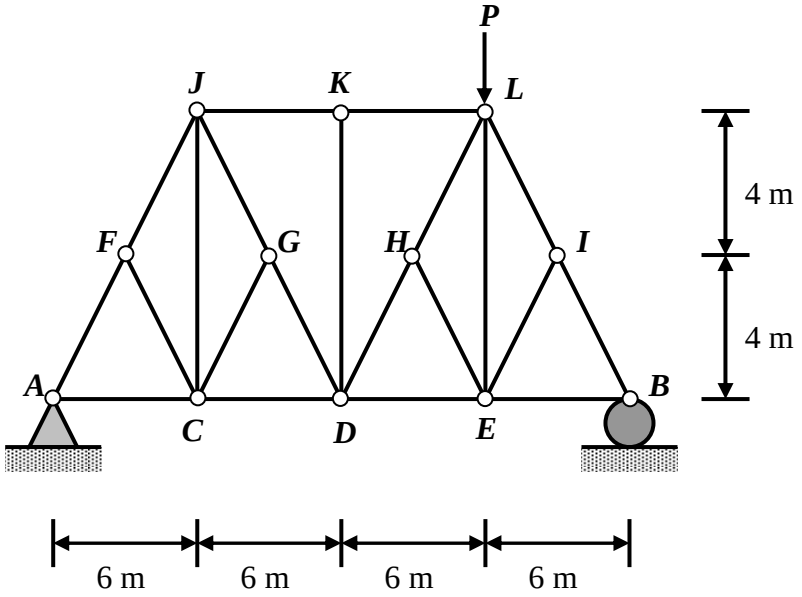
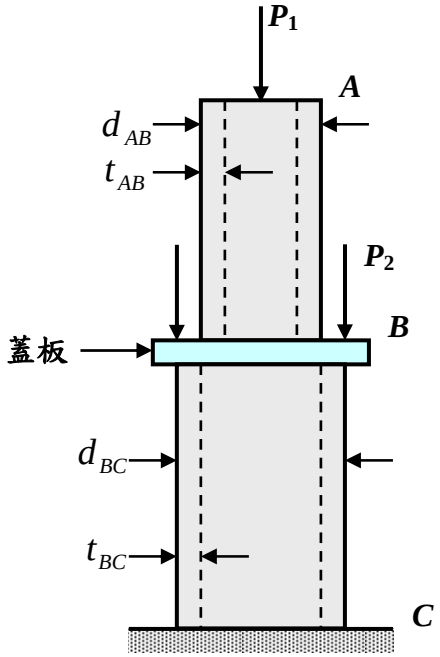
國立臺灣海洋大學河海工程研究所 2017 材料力學 2AB 班第一次大考參考解答

日期：2017 年 04 月 18 日　　姓名：＿＿＿＿＿＿　　學號：＿＿＿＿＿＿

1. (1) 試問下圖哪幾根桿件為零力構件並求 *KL* 桿件之力量大小為何?? (12%)
   (2) 若各桿的截面積為 900 mm^2，且允許承載的平均正應力不能超過 250 MPa，試求最大負載 *P*。 (8%)



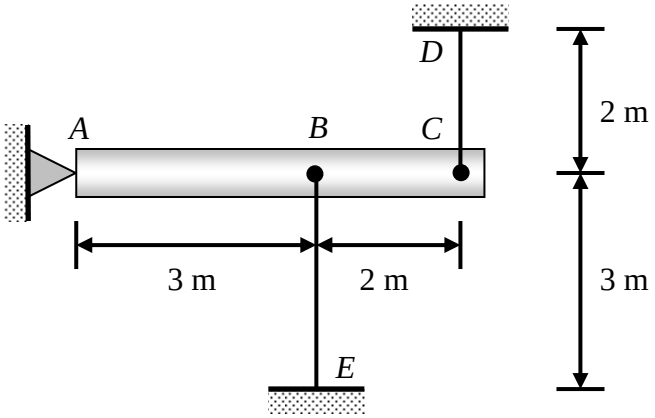
2. 一中空圓形銅管 *ABC* 如下圖所示，支撐作用於頂端的載重 $P_1 = 118$ kN，第二載重 $P_2 = 98$ kN 則均勻分佈於 *B* 處的蓋板上。管上、下部的直徑與壁厚分別為 $d_{AB} = 31$ mm，$t_{AB} = 12$ mm，$d_{BC} = 57$ mm，$t_{BC} = 9$ mm。彈性模數為 $E = 96$ GPa。當兩項載重完全施加後，*BC* 管的壁厚增加了 5×10^{-3} mm；
   (1) 求 *BC* 段內直徑的增加量。 (6%)
   (2) 求銅的蒲松比。 (6%)
   (3) 求 *AB* 段管壁厚的增加量，及 *AB* 內直徑的增加量。 (8%)



3. 如下圖所示，質量可忽略不記的水平剛性桿件 *ABC* 與兩桿相連結。假設該系統一開始處於無應力狀態。若欲在黃銅桿(*CD*)中產生 90 MPa 的拉應力，試求溫度的變化量。(假設兩桿都受到相同的溫度變化) (15%)

(*BE* 桿: $A_{Cu}=1500\,\text{mm}^2$, $E_{Cu}=120\,\text{GPa}$, $\alpha_{Cu}=16.8\times10^{-6}/°C$)

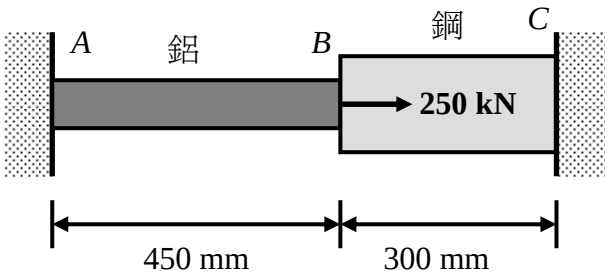
(*CD* 桿: $A_{Br}=1200\,\text{mm}^2$, $E_{Br}=100\,\text{GPa}$, $\alpha_{Br}=18.7\times10^{-6}/°C$)



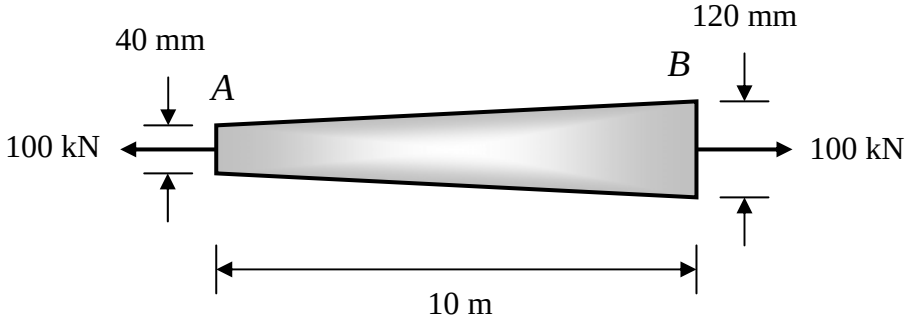
4. 一複合桿件兩端受到如圖所示的剛性支撐。桿件在 30°*C* 時處於無應力狀態。當 250 kN 的負荷作用及溫度上升至 50°*C* 後，試求桿件中每種材料之應力。

($\alpha_{鋼}=12\times10^{-6}/°C$, $A_{鋼}=2700\,\text{mm}^2$, $E_{鋼}=200\,\text{GPa}$) (25%)

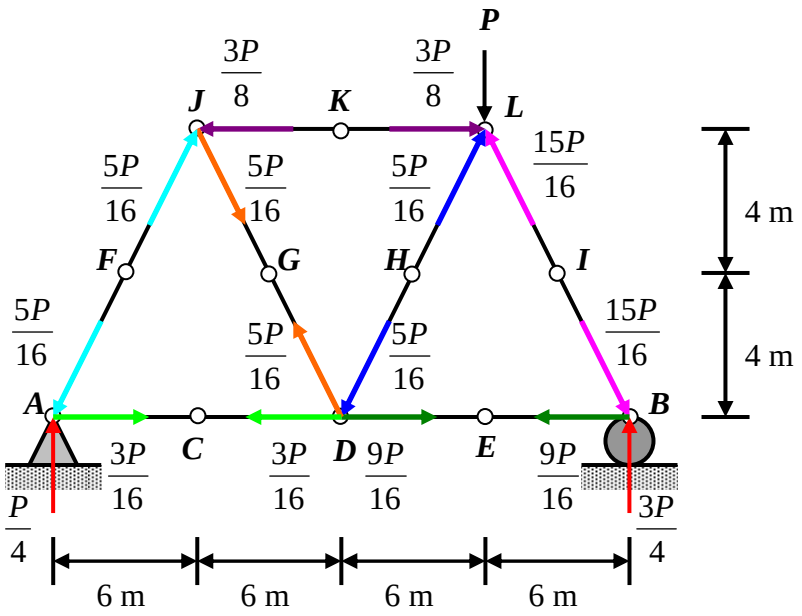
($\alpha_{鋁}=23.1\times10^{-6}/°C$, $A_{鋁}=1800\,\text{mm}^2$, $E_{鋁}=70\,\text{GPa}$)



5. 一長為 10 m 的平面鋼桿 *AB*，有均勻截面厚度 20 mm，而截面的寬度變化如下圖所示。試求桿件受到 100 kN 的軸向力作用所產生的伸長量。 (20%)
(鋼材: $E=200\,\text{GPa}$)



1. (1) 零力桿件: CF、CG、CJ、DK、EH、EI、EL



$\therefore KL$ 桿件之力量大小為 $\dfrac{3P}{8}$ (拉力)

(2) 桿件最大內力為 $\dfrac{15P}{16}$

$$\therefore\quad \sigma = \frac{F}{A} \quad \Rightarrow 250\cdot 10^{6} = \frac{\dfrac{15P}{16}}{900\cdot 10^{-6}} \quad \Rightarrow P = 240000\ (\text{N}) = 240\ (\text{kN})$$

2. (1) $t_{BC} = 9$ mm, $\quad \Delta t_{BC} = 5\times 10^{-3}$ mm

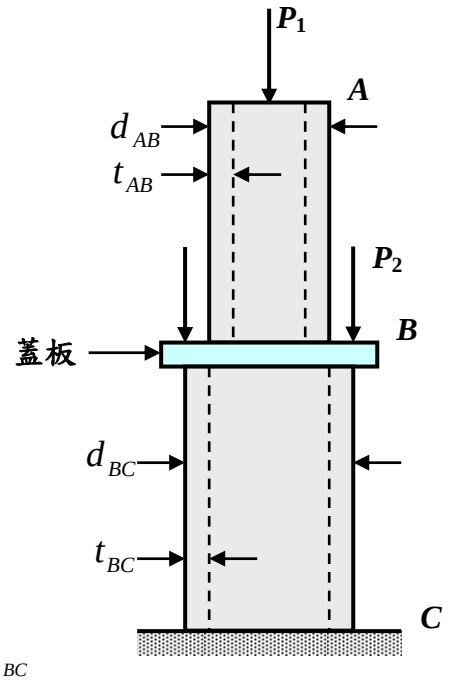
$$(\varepsilon_r)_{BC} = \frac{\Delta t_{BC}}{t_{BC}} = \frac{5\times 10^{-3}}{9} = 5.556\times 10^{-4}$$

BC 內直徑的增加量 $= (\varepsilon_r)_{BC}\cdot(d_{BC} - 2t_{BC})$

$= 5.556\cdot 10^{-4}\cdot(57 - 2\cdot 9)$

$= 2.1668\cdot 10^{-2}$ (mm)

$= 0.021668$ (mm)



(2) $A_{BC} = \dfrac{\pi}{4}\left[d_{BC}^2 - (d_{BC} - 2t_{BC})^2\right]$

$= \dfrac{\pi}{4}\left[(57)^2 - (39)^2\right]$

$= 1357.17\ (\text{mm}^2)$

$= 1.3572\cdot 10^{-3}\ (\text{m}^2)$

$$\sigma_{BC} = E\cdot(\varepsilon_r)_{BC} \quad \Rightarrow \frac{-(118+98)\cdot 10^{3}}{1.3572\cdot 10^{-3}} = 96\cdot 10^{9}\cdot(\varepsilon_\ell)_{BC}$$

$$\Rightarrow (\varepsilon_\ell)_{BC} = -1.6578\cdot 10^{-3}$$

$$\nu = -\frac{(\varepsilon_r)_{BC}}{(\varepsilon_\ell)_{BC}} = \frac{5.556\times 10^{-4}}{1.6578\cdot 10^{-3}} = 0.335$$

(3) $A_{AB} = \dfrac{\pi}{4}\left[d_{AB}^2 - (d_{AB} - 2t_{AB})^2\right]$

$$= \frac{\pi}{4}\left[(31)^2 - (7)^2\right]$$

$$= 716.28 \ \ (\text{mm}^2)$$

$$= 7.1628\cdot 10^{-4} \ \ (\text{m}^2)$$

$$\sigma_{AB} = E\cdot(\varepsilon_\ell)_{AB} \ \Rightarrow \frac{-118\cdot 10^3}{7.1628\cdot 10^{-4}} = 96\cdot 10^9\cdot(\varepsilon_\ell)_{AB}$$

$$\Rightarrow (\varepsilon_\ell)_{AB} = -1.716\cdot 10^{-3}$$

$$\nu = -\frac{(\varepsilon_r)_{AB}}{(\varepsilon_\ell)_{AB}} \Rightarrow (\varepsilon_r)_{AB} = 0.335\cdot 1.716\cdot 10^{-3} = 5.75\cdot 10^{-4}$$

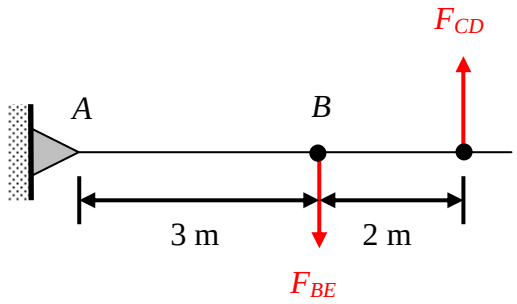
$$\Delta t_{AB} = (\varepsilon_r)_{AB}\cdot t_{AB} = 5.75\cdot 10^{-4}\cdot 12 = 6.9\cdot 10^{-3} \ \ (\text{mm})$$

AB 內直徑的增加量 $= (\varepsilon_r)_{AB}\cdot(d_{AB} - 2t_{AB})$

$$= 5.75\cdot 10^{-4}\cdot(31 - 2\cdot 12)$$

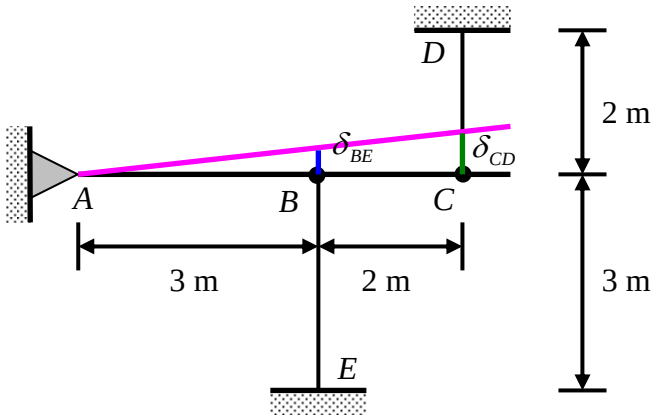
$$= 4.025\cdot 10^{-3} \ \ (\text{mm})$$

3.

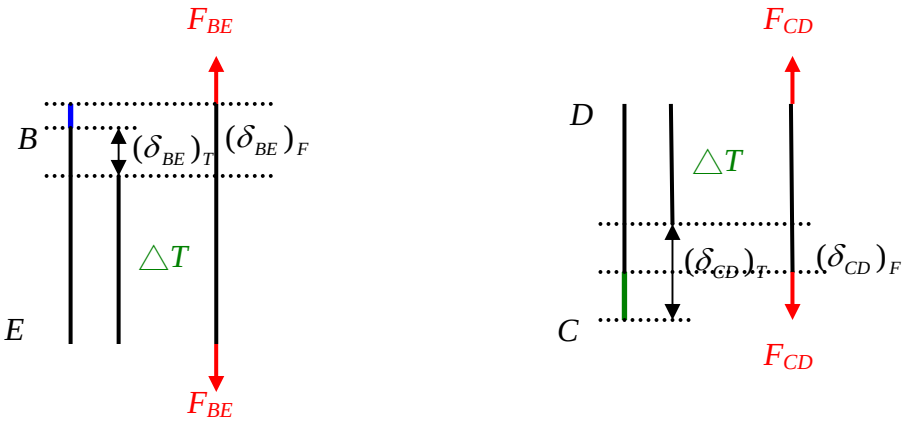


$$F_{CD} = \sigma_{CD}\cdot A_{CD} = 90\cdot 10^6\cdot 1200\cdot 10^{-6} = 108000 \ \ (\text{N}) = 108 \ \ (\text{kN})$$

$$\sum M_A = 0 \quad \Rightarrow F_{BE} = \frac{5}{3}F_{CD} = 180 \ \ (\text{kN})$$



相容方程式：$\delta_{CD} = \frac{5}{3}\delta_{BE}$



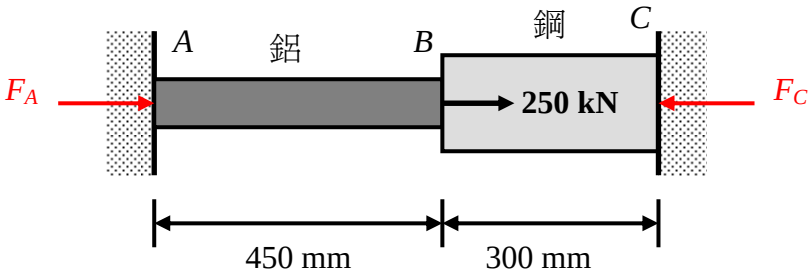
$$\delta_{BE} = (\delta_{BE})_F - (\delta_{BE})_T$$
$$\delta_{CD} = (\delta_{CD})_T - (\delta_{CD})_F$$
$$\therefore (\delta_{CD})_T - (\delta_{CD})_F = \frac{5}{3}[(\delta_{BE})_F - (\delta_{BE})_T]$$

$$\Rightarrow (18.7\cdot 10^{-6}\cdot \Delta T\cdot 2) - (\frac{108\cdot 10^3\cdot 2}{1200\cdot 10^{-6}\cdot 100\cdot 10^9})$$
$$= \frac{5}{3}[\frac{180\cdot 10^3\cdot 3}{1500\cdot 10^{-6}\cdot 120\cdot 10^9} - (16.8\cdot 10^{-6}\cdot \Delta T\cdot 3)]$$

$$\Rightarrow 121.4\cdot 10^{-6}\cdot \Delta T = 0.0068 \qquad \Rightarrow \Delta T = 56\ (^\circ C)$$

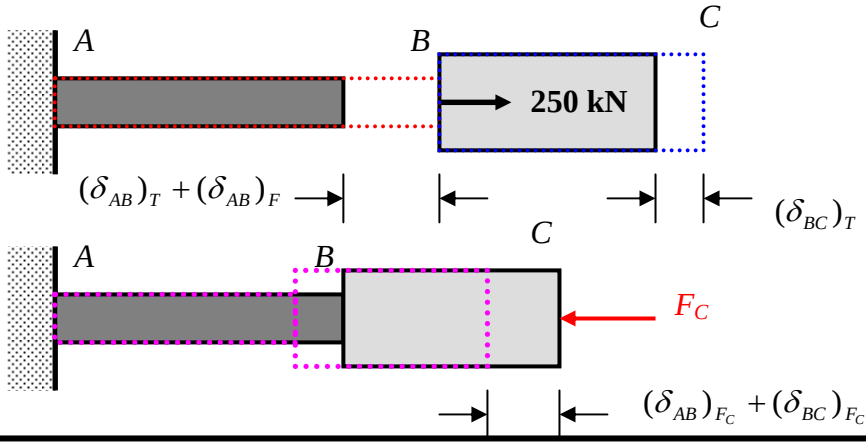
所以為溫度下降 56 °C

4.



$$\sum F_x = 0 \qquad \Rightarrow F_C = F_A + 250$$

力法:



相容方程式：$\delta_{C/A}=0$

$$\Rightarrow \left[(\delta_{AB})_T+(\delta_{AB})_F+(\delta_{BC})_T\right]-\left[(\delta_{AB})_{F_C}+(\delta_{BC})_{F_C}\right]=0$$

$$\Rightarrow \left[(23.1\cdot10^{-6}\cdot20\cdot0.45)+(\frac{250\cdot10^{3}\cdot0.45}{1800\cdot10^{-6}\cdot70\cdot10^{9}})+(12\cdot10^{-6}\cdot20\cdot0.3)\right]$$
$$-\left[\frac{F_c\cdot0.45}{1800\cdot10^{-6}\cdot70\cdot10^{9}}+\frac{F_c\cdot0.3}{2700\cdot10^{-6}\cdot200\cdot10^{9}}\right]=0$$

$$\Rightarrow \left[(23.1\cdot10^{-6}\cdot20\cdot0.45)+(\frac{250\cdot10^{3}\cdot0.45}{1800\cdot10^{-6}\cdot70\cdot10^{9}})+(12\cdot10^{-6}\cdot20\cdot0.3)\right]$$
$$-\left[\frac{F_c\cdot0.45}{1800\cdot10^{-6}\cdot70\cdot10^{9}}+\frac{F_c\cdot0.3}{2700\cdot10^{-6}\cdot200\cdot10^{9}}\right]=0$$

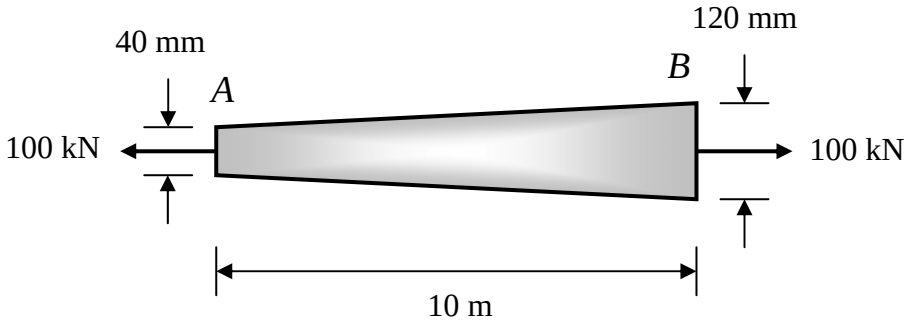
$\Rightarrow F_c=284168$ (N)

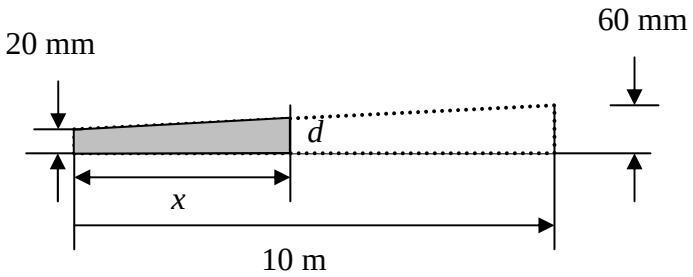
$\Rightarrow F_A=34168$ (N)

$$\sigma_{鋁}=\frac{F_A}{A_{鋁}}=\frac{34168}{1800\cdot10^{-6}}=18.98\cdot10^{6}\ \text{(Pa)}=18.98\ \text{(MPa)}$$

$$\sigma_{鋼}=\frac{F_c}{A_{鋼}}=\frac{284168}{2700\cdot10^{-6}}=18.98\cdot10^{6}\ \text{(Pa)}=105.25\ \text{(MPa)}$$

5.





$$\delta=\int\frac{P}{A(x)\cdot E}dx$$

$$\frac{x}{10}=\frac{d-20}{60-20}\ \Rightarrow d=4x+20$$

$$A(x)=2d\cdot20=160x+800\ \ (\text{mm}^2)$$

$$\delta = \int \frac{P}{A(x)\cdot E}\,dx = \int_0^{10} \frac{100\cdot 10^3}{(160x+800)\cdot 10^{-6}\cdot 200\cdot 10^9}\,dx$$

$$= \frac{1}{320}\int_0^{10} \frac{1}{x+5}\,dx$$

$$= \frac{1}{320}\ln(x+5)\,|_0^{10}$$

$$= 3.433\cdot 10^{-3} \quad \text{(m)} = 3.433 \quad \text{(mm)}$$