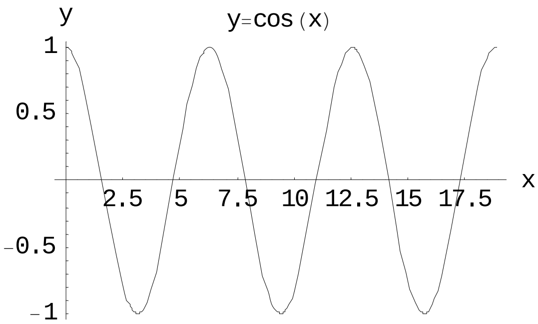
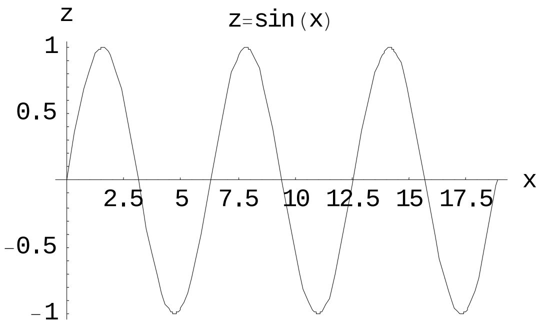
國立臺灣海洋大學河海工程學系 2026 工程數學(二)2AB 班課後練習 03 參考解答

系級：______________　　學號：____________　　姓名：___________

1. 試證明: $\kappa = \dfrac{|\vec{r}' \times \vec{r}''|}{(\vec{r}' \cdot \vec{r}')^{\frac{3}{2}}}$ 與 $\tau = \dfrac{[\,\vec{r}'\ \vec{r}''\ \vec{r}''']}{|\vec{r}' \times \vec{r}''|^2}$ ，其中 $\vec{r}' = \dfrac{d\vec{r}(t)}{dt}$ 。

2. 已知一曲線之位置向量為 $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$ 。若一 3 維曲線投影於 x-y 平面及 x-z 如圖所示，試以 $x(t) = t$ 作為參數，求此曲線之單位切向量、單位法向量與曲率 κ 。





3. 對於兩曲面 $x^2 + y^2 = 1$ 與 $x^2 - y^2 = z$ 試求其交線上任一點之曲率與扭率。

4. 試求曲線 $(x-1)(y-2) = 3$ 上任一點之曲率 κ 與扭率 τ 。

5. 已知 $\kappa(s) = \dfrac{2}{5}$ 與 $\tau(s) = \dfrac{1}{5}$ 並且知道在點 $(2, 0, 0)$ 其 $\vec{T}(0) = (0, \dfrac{2}{\sqrt{5}}, \dfrac{1}{\sqrt{5}})$、$\vec{N}(0) = (-1, 0, 0)$ 與 $\vec{B}(s) = (0, -\dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}})$，試問該曲線表示式為何 ？

參考解答:

1. (1) $$\frac{d\vec{r}(t)}{ds} = \frac{d\vec{r}(t)}{dt} \cdot \frac{dt}{ds} = \frac{\vec{r}'(t)}{\sqrt{\vec{r}'(t) \cdot \vec{r}'(t)}}$$

$$\frac{d^2\vec{r}(t)}{ds^2} = \frac{d}{dt}\left(\frac{d\vec{r}(t)}{ds}\right) \cdot \frac{dt}{ds} = \frac{1}{\sqrt{\vec{r}'(t) \cdot \vec{r}'(t)}} \left[\frac{\vec{r}''(t)}{\sqrt{\vec{r}'(t) \cdot \vec{r}'(t)}} - \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{(\vec{r}' \cdot \vec{r}')^{\frac{3}{2}}} \vec{r}'(t) \right]$$

$$= \left[\frac{\vec{r}''(t)}{\vec{r}'(t) \cdot \vec{r}'(t)} - \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{(\vec{r}' \cdot \vec{r}')^2} \vec{r}'(t) \right]$$

$$\kappa^2 = \frac{d^2\vec{r}(t)}{ds^2} \cdot \frac{d^2\vec{r}(t)}{ds^2} = \left[\frac{\vec{r}''(t)}{\vec{r}'(t) \cdot \vec{r}'(t)} - \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{(\vec{r}' \cdot \vec{r}')^2} \vec{r}'(t) \right] \cdot \left[\frac{\vec{r}''(t)}{\vec{r}'(t) \cdot \vec{r}'(t)} - \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{(\vec{r}' \cdot \vec{r}')^2} \vec{r}'(t) \right]$$

$$= \frac{[\vec{r}'(t) \cdot \vec{r}'(t)][\vec{r}''(t) \cdot \vec{r}''(t)] - [\vec{r}'(t) \cdot \vec{r}''(t)]^2}{[\vec{r}'(t) \cdot \vec{r}'(t)]^3}$$

由 Lagrange Identity: $\left|\vec{A}\times\vec{B}\right| = \sqrt{(\vec{A}\cdot\vec{A})(\vec{B}\cdot\vec{B})-(\vec{A}\cdot\vec{B})^2}$ 可知

$$\kappa^2 = \frac{\left|\vec{r}\,'(t)\times\vec{r}\,''(t)\right|^2}{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]^3} \quad \Rightarrow \kappa = \frac{\left|\vec{r}\,'(t)\times\vec{r}\,''(t)\right|}{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]^{\frac{3}{2}}}$$

(2) $\vec{t}(t) = \dfrac{1}{\sqrt{\vec{r}\,'(t)\cdot\vec{r}\,'(t)}}\vec{r}\,'(t)$

$$\vec{n}(t) = \frac{1}{\kappa}\frac{d^2\vec{r}(t)}{ds^2} = \frac{1}{\kappa}\frac{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]\,\vec{r}\,''(t)-[\vec{r}\,'(t)\cdot\vec{r}\,''(t)]\vec{r}\,'(t)}{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]^2}$$

又 $\vec{b}(t) = \vec{t}(t)\times\vec{n}(t) = \dfrac{1}{\kappa}\dfrac{\vec{r}\,'(t)\times\vec{r}\,''(t)}{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]^{\frac{3}{2}}} = \dfrac{\vec{r}\,'(t)\times\vec{r}\,''(t)}{\left|\vec{r}\,'(t)\times\vec{r}\,''(t)\right|}$

令 $\phi(t) = \left|\vec{r}\,'(t)\times\vec{r}\,''(t)\right|$

$$\frac{d\vec{b}(t)}{ds} = \frac{d\vec{b}(t)}{dt}\cdot\frac{dt}{ds} = \frac{1}{\sqrt{\vec{r}\,'(t)\cdot\vec{r}\,'(t)}}\left[\frac{\vec{r}\,'(t)\times\vec{r}\,'''(t)}{\phi(t)} - \frac{\phi'(t)}{[\phi(t)]^2}(\vec{r}\,'(t)\times\vec{r}\,''(t))\right]$$

由 $\tau = \vec{n}(t)\cdot\dfrac{d\vec{b}(t)}{ds}$ 可得

$$\tau = \frac{1}{\kappa}\frac{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]\,\vec{r}\,''(t)-[\vec{r}\,'(t)\cdot\vec{r}\,''(t)]\vec{r}\,'(t)}{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]^2}\cdot\frac{1}{\sqrt{\vec{r}\,'(t)\cdot\vec{r}\,'(t)}}\left[\frac{\vec{r}\,'(t)\times\vec{r}\,'''(t)}{\phi(t)} - \frac{\phi'(t)}{[\phi(t)]^2}(\vec{r}\,'(t)\times\vec{r}\,''(t))\right]$$

$$= \frac{1}{\kappa}\frac{-[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]\,\vec{r}\,''(t)}{[\vec{r}\,'(t)\cdot\vec{r}\,'(t)]^2}\cdot\frac{\vec{r}\,'(t)\times\vec{r}\,'''(t)}{\phi(t)\sqrt{\vec{r}\,'(t)\cdot\vec{r}\,'(t)}}$$

$$= \frac{[\vec{r}\,'\;\vec{r}\,''\;\vec{r}\,''']}{\left|\vec{r}\,'\times\vec{r}\,''\right|^2}$$

2. $x = t$， $y = \cos x = \cos t$， $z = \sin x = \sin t$

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} = t\vec{i} + \cos t\,\vec{j} + \sin t\,\vec{k} \quad \Rightarrow \vec{r}\,'(t) = \vec{i} - \sin t\,\vec{j} + \cos t\,\vec{k}$$

單位切向量 $\vec{t}(t) = \dfrac{t\vec{i} - \sin t\,\vec{j} + \cos t\,\vec{k}}{\left|t\vec{i} - \sin t\,\vec{j} + \cos t\,\vec{k}\right|} = \dfrac{1}{\sqrt{2}}(\vec{i} - \sin t\,\vec{j} + \cos t\,\vec{k})$

法向量 $\vec{t}\,'(t) = \dfrac{1}{\sqrt{2}}(-\cos t\,\vec{j} - \sin t\,\vec{k})$

單位法向量 $\vec{n}(t) = -\cos t\,\vec{j} - \sin t\,\vec{k}$

曲率 $\kappa = \left|\dfrac{\vec{t}\,'}{\vec{r}\,'(t)}\right| = \dfrac{1}{2}$

3. 兩曲面交線可由 $x^2+y^2=1$ 可得 $x=\cos t$ 與 $y=\cos t$ 帶入 $x^2-y^2=z$ 可得 $z=\cos^2 t-\sin^2 t=\cos 2t$

$\therefore$ 兩曲面交線可由參數式表示，即 $\vec{r}(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+\cos 2t\,\hat{k}$

$$\vec{r}\,'(t)=-\sin t\,\hat{i}+\cos t\,\hat{j}-2\sin 2t\,\hat{k}$$

$$\left|\vec{r}\,'(t)\right|=(1+4\sin^2 2t)^{\frac{1}{2}}=(3-2\cos 4t)^{\frac{1}{2}}$$

$$\vec{t}(t)=\frac{\vec{r}\,'(t)}{\left|\vec{r}\,'(t)\right|}=\frac{-\sin t\,\hat{i}+\cos t\,\hat{j}-2\sin 2t\,\hat{k}}{(1+4\sin^2 2t)^{\frac{1}{2}}}$$

$$\vec{t}\,'(t)=-\frac{3\cos t-3\cos 3t+\cos 5t}{(1+4\sin^2 2t)^{\frac{3}{2}}}\hat{i}-\frac{3\sin t+3\sin 3t+\sin 5t}{(1+4\sin^2 2t)^{\frac{3}{2}}}\hat{j}-\frac{4\cos 2t}{(1+4\sin^2 2t)^{\frac{3}{2}}}\hat{k}$$

$$\left|\vec{t}\,'(t)\right|=\frac{\sqrt{11+6\cos 4t}}{1+4\sin^2 2t}$$

$$\kappa=\frac{\left|\vec{t}\,'(t)\right|}{\left|\vec{r}\,'(t)\right|}=\frac{\frac{\sqrt{11+6\cos 4t}}{1+4\sin^2 2t}}{(1+4\sin^2 2t)^{\frac{1}{2}}}=\frac{\sqrt{11+6\cos 4t}}{(1+4\sin^2 2t)^{\frac{3}{2}}}=\frac{\sqrt{5+12z^2}}{[1+4(1-z^2)]^{\frac{3}{2}}}$$

$$\vec{n}(t)=\frac{\vec{t}\,'(t)}{\left|\vec{t}\,'(t)\right|}=-\frac{3\cos t-3\cos 3t+\cos 5t}{(1+4\sin^2 2t)^{\frac{1}{2}}\sqrt{11+6\cos 4t}}\,\hat{i}-\frac{3\sin t+3\sin 3t+\sin 5t}{(1+4\sin^2 2t)^{\frac{1}{2}}\sqrt{11+6\cos 4t}}\,\hat{j}$$
$$-\frac{4\cos 2t}{(1+4\sin^2 2t)^{\frac{1}{2}}\sqrt{11+6\cos 4t}}\hat{k}$$

$$\vec{t}(t)\times\vec{n}(t)=\frac{-4\cos^3 t}{\sqrt{11+6\cos 4t}}\,\hat{i}+\frac{4\sin^3 t}{\sqrt{11+6\cos 4t}}\,\hat{j}+\frac{1}{\sqrt{11+6\cos 4t}}\hat{k}$$

$$\frac{d(\vec{t}(t)\times\vec{n}(t))}{dt}=\frac{-12\cos^2 t(11\sin t-5\sin 3t+\sin 5t)}{(11+6\cos 4t)^{\frac{3}{2}}}\,\hat{i}$$
$$+\frac{12\sin^2 t(11\cos t+5\cos 3t+\cos 5t)}{(11+6\cos 4t)^{\frac{3}{2}}}\,\hat{j}+\frac{12\sin 4t}{(11+6\cos 4t)^{\frac{3}{2}}}\hat{k}$$

$$\left|\frac{d(\vec{t}(t)\times\vec{n}(t))}{dt}\right|=\frac{6\sin 2t\sqrt{(1+4\sin^2 2t)}}{11+6\cos 4t}$$

$$\tau=\left|\frac{d\vec{B}(s)}{ds}\right|=\frac{d\left|\vec{b}(t)\right|}{ds}=\frac{dt}{ds}\frac{d\left|\vec{t}(t)\times\vec{n}(t)\right|}{dt}=\frac{1}{\left|\vec{r}\,'(t)\right|}\left|\frac{d(\vec{t}(t)\times\vec{n}(t))}{dt}\right|$$
$$=\frac{6\sin 2t}{11+6\cos 4t}=\frac{12xy}{5+12z^2}$$

另解: $\vec{r}'(t) = -\sin t\,\hat{i} + \cos t\,\hat{j} - 2\sin 2t\,\hat{k}$

$$\vec{r}''(t) = -\cos t\,\hat{i} - \sin t\,\hat{j} - 4\cos 2t\,\hat{k}$$

$$\vec{r}'''(t) = \sin t\,\hat{i} - \cos t\,\hat{j} + 8\sin 2t\,\hat{k}$$

$$\vec{r}'(t)\cdot\vec{r}'(t) = 1 + 4\sin^2 2t$$

$$\begin{aligned}
&\vec{r}'(t)\times\vec{r}''(t)\\
&= (-4\cos t\cos 2t - 2\sin t\sin 2t)\,\hat{i} + (2\sin 2t\cos t - 4\sin t\cos 2t)\,\hat{j} + \hat{k}\\
&= (-4\cos t\cos 2t - 4\sin^2 t\cos t)\,\hat{i} + (4\sin t\cos^2 t - 4\sin t\cos 2t)\,\hat{j} + \hat{k}\\
&= [-4\cos t\cos 2t - 2(1-\cos 2t)\cos t]\,\hat{i} + [2\sin t(1+\cos 2t) - 4\sin t\cos 2t]\,\hat{j} + \hat{k}\\
&= (-4\cos t\cos 2t - 2\cos t + 2\cos 2t\cos t)\,\hat{i} + (2\sin t + 2\sin t\cos 2t - 4\sin t\cos 2t)\,\hat{j} + \hat{k}\\
&= (-2\cos t - 2\cos t\cos 2t)\,\hat{i} + (2\sin t - 2\sin t\cos 2t)\,\hat{j} + \hat{k}
\end{aligned}$$

$$\vec{r}'(t)\cdot[\vec{r}''(t)\times\vec{r}'''(t)] = \begin{vmatrix} -\sin t & \cos t & -2\sin 2t \\ -\cos t & -\sin t & -4\cos 2t \\ \sin t & -\cos t & 8\sin 2t \end{vmatrix} = 6\sin 2t$$

$$\kappa = \frac{\left|\vec{r}'(t)\times\vec{r}''(t)\right|}{\left[\vec{r}'(t)\cdot\vec{r}'(t)\right]^{\frac{3}{2}}} = \frac{\sqrt{5+12\cos^2 2t}}{(1+4\sin^2 2t)^{\frac{3}{2}}} = \frac{\sqrt{5+12z^2}}{[1+4(1-z^2)]^{\frac{3}{2}}}$$

$$\tau = \frac{[\vec{r}'(t)\ \vec{r}''(t)\ \vec{r}'''(t)]}{\left|\vec{r}'(t)\times\vec{r}''(t)\right|^2} = \frac{6\sin 2t}{5+12\cos^2 2t} = \frac{12xy}{5+12z^2}$$

4. $(x-1)(y-2) = 3 \ \Rightarrow y = \dfrac{3}{(x-1)} + 2$

$$\Rightarrow y' = \frac{-3}{(x-1)^2}$$

$$\Rightarrow y'' = \frac{6}{(x-1)^3}$$

$$\kappa = \frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}} = \frac{\left|\dfrac{6}{(x-1)^3}\right|}{\left[1+\dfrac{9}{(x-1)^4}\right]^{\frac{3}{2}}} = \frac{6\left|(x-1)^3\right|}{\left[(x-1)^4+9\right]^{\frac{3}{2}}}$$

因為是平面曲線，所以$\tau = 0$

5. 由 Frenet 定理可知：

$$\frac{d}{ds}\begin{Bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{Bmatrix}=\begin{bmatrix}0 & \kappa(s) & 0\\ -\kappa(s) & 0 & \tau(s)\\ 0 & -\tau(s) & 0\end{bmatrix}\begin{Bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{Bmatrix}$$

令 $P(s)=\begin{Bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{Bmatrix} \quad \Rightarrow \vec{P}'(s)=A(s)\vec{P}(s)$

$$A=\begin{bmatrix}0 & \frac{2}{5} & 0\\ -\frac{2}{5} & 0 & \frac{1}{5}\\ 0 & -\frac{1}{5} & 0\end{bmatrix}$$

$$\det(A-\lambda I)=\begin{vmatrix}-\lambda & \frac{2}{5} & 0\\ -\frac{2}{5} & -\lambda & \frac{1}{5}\\ 0 & -\frac{1}{5} & -\lambda\end{vmatrix}=0 \quad \Rightarrow \lambda^3+\frac{\lambda}{5}=0 \quad \Rightarrow \lambda=0,\ \lambda=\pm\frac{i}{\sqrt{5}}$$

$$\lambda_1=0 \quad \Rightarrow v_1=\begin{Bmatrix}1\\ 0\\ 2\end{Bmatrix}$$

$$\lambda_2=i \quad \Rightarrow v_2=\begin{Bmatrix}\frac{2}{\sqrt{5}}\\ i\\ -\frac{1}{\sqrt{5}}\end{Bmatrix}$$

$$\lambda_3=-i \quad \Rightarrow v_3=\begin{Bmatrix}-\frac{2}{\sqrt{5}}\\ i\\ \frac{1}{\sqrt{5}}\end{Bmatrix}$$

$$Q=\begin{bmatrix}1 & \frac{2}{\sqrt{5}} & -\frac{2}{\sqrt{5}}\\ 0 & i & i\\ 2 & -\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{bmatrix},\quad D=\begin{bmatrix}0 & 0 & 0\\ 0 & \frac{i}{\sqrt{5}} & 0\\ 0 & 0 & -\frac{i}{\sqrt{5}}\end{bmatrix},\quad Q^{-1}=\begin{bmatrix}\frac{1}{5} & 0 & \frac{2}{5}\\ \frac{1}{\sqrt{5}} & -\frac{i}{2} & -\frac{1}{2\sqrt{5}}\\ -\frac{1}{\sqrt{5}} & -\frac{i}{2} & \frac{1}{2\sqrt{5}}\end{bmatrix}$$

$$\vec{P}(s)=e^{As}\cdot\vec{P}(0)$$

$$=Qe^{Ds}Q^{-1}\begin{Bmatrix} 0\vec{i}+\frac{2}{\sqrt{5}}\vec{j}+\frac{1}{\sqrt{5}}\vec{k} \\ -\vec{i}+0\,\vec{j}+0\vec{k} \\ 0\vec{i}-\frac{1}{\sqrt{5}}\vec{j}+\frac{2}{\sqrt{5}}\vec{k} \end{Bmatrix}$$

$$=\begin{bmatrix} \frac{1}{5}+\frac{2}{5}(e^{-\frac{is}{\sqrt{5}}}+e^{\frac{is}{\sqrt{5}}}) & \frac{i}{\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}}-e^{\frac{is}{\sqrt{5}}}) & \frac{2}{5}-\frac{1}{5}(e^{-\frac{is}{\sqrt{5}}}+e^{\frac{is}{\sqrt{5}}}) \\ -\frac{i}{\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}}-e^{\frac{is}{\sqrt{5}}}) & \frac{1}{2}(e^{-\frac{is}{\sqrt{5}}}+e^{\frac{is}{\sqrt{5}}}) & \frac{i}{2\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}}-e^{\frac{is}{\sqrt{5}}}) \\ \frac{2}{5}-\frac{1}{5}(e^{-\frac{is}{\sqrt{5}}}+e^{\frac{is}{\sqrt{5}}}) & -\frac{i}{2\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}}-e^{\frac{is}{\sqrt{5}}}) & \frac{4}{5}+\frac{1}{10}(e^{-\frac{is}{\sqrt{5}}}+e^{\frac{is}{\sqrt{5}}}) \end{bmatrix}\begin{Bmatrix} \frac{1}{\sqrt{5}}(2\vec{j}+\vec{k}) \\ -\vec{i} \\ -\frac{1}{\sqrt{5}}(\vec{j}-2\vec{k}) \end{Bmatrix}$$

$$=\begin{bmatrix} \frac{1}{5}+\frac{4}{5}\cos(\frac{s}{\sqrt{5}}) & \frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) & \frac{2}{5}-\frac{2}{5}\cos(\frac{s}{\sqrt{5}}) \\ -\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) & \cos(\frac{s}{\sqrt{5}}) & \frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) \\ \frac{2}{5}-\frac{2}{5}\cos(\frac{s}{\sqrt{5}}) & -\frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) & \frac{4}{5}+\frac{1}{5}\cos(\frac{s}{\sqrt{5}}) \end{bmatrix}\begin{Bmatrix} \frac{1}{\sqrt{5}}(2\vec{j}+\vec{k}) \\ -\vec{i} \\ -\frac{1}{\sqrt{5}}(\vec{j}-2\vec{k}) \end{Bmatrix}$$

$$=\begin{Bmatrix} \frac{1}{5\sqrt{5}}[1+4\cos(\frac{s}{\sqrt{5}})]\cdot(2\vec{j}+\vec{k})-\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\cdot(\vec{i})-\frac{2}{5\sqrt{5}}[1-\cos(\frac{s}{\sqrt{5}})]\cdot(\vec{j}-2\vec{k}) \\ -\frac{2}{5}\sin(\frac{s}{\sqrt{5}})\cdot(2\vec{j}+\vec{k})-\cos(s)\cdot(\vec{i})-\frac{1}{5}\sin(\frac{s}{\sqrt{5}})\cdot(\vec{j}-2\vec{k}) \\ \frac{2}{5\sqrt{5}}[1-\cos(\frac{s}{\sqrt{5}})]\cdot(2\vec{j}+\vec{k})+\frac{1}{\sqrt{5}}\sin(s)\cdot(\vec{i})-\frac{1}{5\sqrt{5}}[4+\cos(\frac{s}{\sqrt{5}})]\cdot(\vec{j}-2\vec{k}) \end{Bmatrix}$$

$$=\begin{Bmatrix} -\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}+\frac{2}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{1}{\sqrt{5}}\vec{k} \\ -\cos(\frac{s}{\sqrt{5}})\,\vec{i}-\sin(\frac{s}{\sqrt{5}})\,\vec{j} \\ \frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}-\frac{1}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{2}{\sqrt{5}}\vec{k} \end{Bmatrix}$$

所以可得

$$\vec{T}(s)=-\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}+\frac{2}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{1}{\sqrt{5}}\vec{k}$$

$$\vec{N}(s)=-\cos(\frac{s}{\sqrt{5}})\,\vec{i}-\sin(\frac{s}{\sqrt{5}})\,\vec{j}$$

$$\vec{B}(s)=\frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}-\frac{1}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{2}{\sqrt{5}}\vec{k}$$

由於 $\frac{d\vec{R}(s)}{ds}=\vec{T}(s)=-\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}+\frac{2}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{1}{\sqrt{5}}\vec{k}$

$$\Rightarrow \vec{R}(s)=[2\cos(\frac{s}{\sqrt{5}})+c_1]\vec{i}+[2\sin(\frac{s}{\sqrt{5}})+c_2]\,\vec{j}+[\frac{s}{\sqrt{5}}+c_3]\vec{k}$$

又 $X(0)=2$，$Y(0)=0$ 與 $Z(0)=0$

$$X(s)=2\cos(\frac{s}{\sqrt{5}})+c_1,\quad X(0)=2 \quad \Rightarrow c_1=0$$

$$Y(s)=2\sin(\frac{s}{\sqrt{5}})+c_2,\quad Y(0)=0 \quad \Rightarrow c_2=0$$

$$Z(s)=\frac{s}{\sqrt{5}}+c_3,\quad Z(0)=0 \quad \Rightarrow c_3=0$$

$$\therefore \quad \vec{R}(s)=2\cos(\frac{s}{\sqrt{5}})\vec{i}+2\sin(\frac{s}{\sqrt{5}})\vec{j}+\frac{s}{\sqrt{5}}\vec{k}$$