

系級：_____

學號：_____

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$$1. \mathbf{A} = \begin{bmatrix} 1 & 5 & 1 \\ 3 & -2 & 5 \\ 4 & 0 & 6 \end{bmatrix}$$

(1) 試將 $\mathbf{A}$ 化成對稱矩陣 $\mathbf{R}$ 及反對稱矩陣 $\mathbf{S}$ 。(2) 試求 $\det(\mathbf{A}) = ?$ (3) 試求 $\mathbf{A}^{-1} = ?$

$$2. \mathbf{A} = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 2 & 0 & 2 \\ 4 & 1 & -1 & -1 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

(1) 試求 $\det(\mathbf{A}) = ?$ (2) 試求 $\mathbf{A}^{-1} = ?$

$$3. \text{給一聯立方程組} \begin{cases} x_1 - x_2 + 3x_3 - 3x_4 = 3 \\ -5x_1 + 2x_2 - 5x_3 + 4x_4 = -5 \\ -3x_1 - 4x_2 + 7x_3 - 2x_4 = 7 \\ 2x_1 + 3x_2 + x_3 + 5x_4 = 1 \end{cases}$$

(1) 試以高斯消去法解之。

(2) 試以線性代數系統 $\mathbf{Ax} = \mathbf{b}$ 解之。

參考解答：

$$1. (1) \mathbf{R} = \frac{1}{2}(\mathbf{A} + \mathbf{A}^T) = \begin{bmatrix} 1 & 4 & \frac{5}{2} \\ 4 & -2 & \frac{5}{2} \\ \frac{5}{2} & \frac{5}{2} & 6 \end{bmatrix}, \quad \mathbf{S} = \frac{1}{2}(\mathbf{A} - \mathbf{A}^T) = \begin{bmatrix} 0 & 1 & -\frac{3}{2} \\ -1 & 0 & \frac{5}{2} \\ \frac{3}{2} & -\frac{5}{2} & 0 \end{bmatrix}$$

(2) $\det(\mathbf{A}) = 6$

$$(3) \mathbf{A}^{-1} = \begin{bmatrix} -2 & -5 & \frac{9}{2} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{4}{3} & \frac{10}{3} & -\frac{17}{6} \end{bmatrix}$$

2. (1) $\det(\mathbf{A})=128$

$$(2) \mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{8} & -\frac{1}{16} & \frac{1}{4} & 0 \\ -\frac{1}{4} & \frac{1}{16} & -\frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & -\frac{3}{16} & 0 & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & -\frac{1}{8} & -\frac{1}{8} \end{bmatrix}$$

$$3. \mathbf{A} = \begin{bmatrix} 1 & -1 & 3 & -3 \\ -5 & 2 & -5 & 4 \\ -3 & -4 & 7 & -2 \\ 2 & 3 & 1 & 5 \end{bmatrix}, \mathbf{b} = \begin{Bmatrix} 3 \\ -5 \\ 7 \\ 1 \end{Bmatrix}$$

$$(1) \left[\begin{array}{cccc|c} 1 & -1 & 3 & -3 & 3 \\ -5 & 2 & -5 & 4 & -5 \\ -3 & -4 & 7 & -2 & 7 \\ 2 & 3 & 1 & 5 & 1 \end{array} \right] \longrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -3 & 3 \\ 0 & -3 & 10 & -11 & 10 \\ 0 & -7 & 16 & -11 & 16 \\ 0 & 5 & -5 & 11 & -5 \end{array} \right]$$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -3 & 3 \\ 0 & -3 & 10 & -11 & 10 \\ 0 & 0 & -\frac{22}{3} & \frac{44}{3} & -\frac{22}{3} \\ 0 & 0 & \frac{35}{3} & -\frac{22}{3} & \frac{35}{3} \end{array} \right] \longrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -3 & 3 \\ 0 & -3 & 10 & -11 & 10 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 35 & -22 & 35 \end{array} \right]$$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -3 & 3 \\ 0 & -3 & 10 & -11 & 10 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 48 & 0 \end{array} \right]$$

$$\Rightarrow x_4 = 0, x_3 = 1, x_2 = 0, x_1 = 0$$

$$(2) \mathbf{Ax} = \mathbf{b} \Rightarrow \mathbf{x} = \mathbf{A}^{-1}\mathbf{b} = \begin{bmatrix} -\frac{3}{11} & -\frac{5}{22} & -\frac{1}{22} & 0 \\ \frac{7}{8} & \frac{11}{32} & -\frac{5}{32} & \frac{3}{16} \\ \frac{1}{4} & \frac{1}{16} & \frac{1}{16} & \frac{1}{8} \\ -\frac{41}{88} & -\frac{45}{352} & \frac{35}{352} & \frac{1}{16} \end{bmatrix} \begin{Bmatrix} 3 \\ -5 \\ 7 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}$$