

系級：_____ 學號：_____ 姓名：_____

1. 試以參數變異法求解下述微分方程式：

(1) $y'' - 4y = 8x^2 - 2x$

(2) $y'' + 4y = \tan 2x$

(3) $y'' - 6y' + 9y = x^{-2}e^{3x}$

2. 試解下述微分方程：

(1) $x^3 y''' + x^2 y'' - 2xy' + 2y = x^{-2}$

(2) $x^2 y'' - 3xy' + 4y = 3x^2$

(3) $xy'' - y' + 2x^{-1}y = 2\cos(\ln x)$

(4) $x^2 y'' - 2xy' + 2y = x \ln x$

(5) $x^2 y'' - 4xy' + 6y = -7x^4 \sin x$

(6) $xy'' - y' = 2x^2 e^x$

(7) $x^2 y'' + xy' + y = \sec(\ln x)$

[參考解答]

1. (1) $y = y_h + y_p = C_1 e^{-2x} + C_2 e^{2x} - 2x^2 + \frac{1}{2}x - 1$

(2) $y = y_h + y_p = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{4} \cos 2x \cdot \ln \left| \tan\left(\frac{\pi}{4} + x\right) \right|$

(3) $y = y_h + y_p = e^{3x} (C_1 + C_2 x - \ln|x|)$

2. (1) $y = y_h + y_p = C_1 x + C_2 x^{-1} + C_3 x^2 - \frac{1}{12} x^{-2}$

(2) $y = y_h + y_p = C_1 x^2 + C_2 x^2 \cdot \ln x + \frac{3}{2} x^2 (\ln x)^2$

(3) $y = y_h + y_p = x[C_1 \cos(\ln x) + C_2 \sin(\ln x)] + x \ln x \cdot \sin(\ln x)$

(4) $y = y_h + y_p = C_1 x + C_2 x^2 - \frac{1}{2} x (\ln x)^2 - x \ln x$

(5) $y = y_h + y_p = C_1 x^2 + C_2 x^3 + 7x^2 \sin x$

(6) $y = y_h + y_p = C_1 + C_2 x^2 + e^x (2x - 2)$

(7) $y = y_h + y_p = C_1 \cos(\ln x) + C_2 \sin(\ln x) + \ln|\cos(\ln x)| \cdot \cos(\ln x) + \ln x \cdot \sin(\ln x)$