

系級：_____ 學號：_____ 姓名：_____

1. 試以全微分法求解下述 ODE:

(1) $xy' - y = \frac{y}{\ln y - \ln x}$

(2) $y(2 + 2x^2y^{\frac{1}{2}})dx + x(x^2y^{\frac{1}{2}} + 2)dy = 0$

(3) $(2x^2y + 3y^3)dx - (x^3 + 2xy^2)dy = 0$

(4) $y' = \frac{y + xy^3(1 + \ln x)}{x}$

(5) $(3xe^y + 2y)dx + (x^2e^y + x)dy = 0$

2. 試以 Picard 法求解

$y' = 2y^2, y(0) = 1$

3. 給一微分方程 $x^2y'' + 2xy' - 12y = 0$, 已知一補解為 $y_1 = x^3$, 試求另一補解 y_2

[參考解答]

1. (1) $\frac{1}{2}[\ln(\frac{y}{x})]^2 = \ln x + C$

(2) $2xy + \frac{2}{3}(x^2y)^{\frac{3}{2}} = C$

(3) $-\frac{1}{2}(x^2y^{-1})^{-2} - \frac{1}{2}(x^3y^{-2})^{-2} = C$

(4) $-\frac{1}{2}(\frac{x}{y})^2 = \frac{2}{9}x^3 + \frac{1}{3}x^3 \ln x + C$

(5) $x^3e^y + x^2y = C$

2. $y = \tan x$

3. $y_2 = x^{-4}$