

系級：_____ 學號：_____ 姓名：_____

1. 試以一階線性 ODE 法求解下述 ODE:

(1) $xy' + 2y = 2e^{x^2}$

(2) $y \sin y - 3xyy' = y$

2. 試求解下述常微分方程式

(1) $y' + \frac{3}{x}y = -2xy^{\frac{5}{2}}$

(2) $y' = y^2 - 2xy + x^2 + 1$

(3) $x^2(y')^2 + 4xyy' + 3y^2 = 0$

3. 試求出下述常微分方程式之通解(general solution)與奇解(singular solution)

$$y = xy' + \frac{1}{2}(y')^2$$

[參考解答]

1. (1) $y = (C + e^{x^2})x^{-2}$

(2) $x = \frac{1}{10}e^{3y}(3\sin y - \cos y) + Ce^{-3y}$

2. (1) $\frac{2}{3}x^{-\frac{9}{2}}y^{-\frac{3}{2}} + \frac{4}{5}x^{-\frac{5}{2}} = C$

(2) $y = x + \frac{1}{C - x}$

(3) $(y - \frac{C}{x^3})(y - \frac{C}{x}) = 0$

3. General solution: $y = Cx + \frac{1}{2}C^2$

Singular solution: $y = -\frac{1}{2}x^2$