國立臺灣海洋大學河海工程學系 2022 工程數學(二) 第三次作業參考解答

系級：＿＿＿＿＿＿＿　　學號：＿＿＿＿＿＿　　姓名：＿＿＿＿＿＿

1. 對於兩曲面 $x^2+y^2=1$ 與 $x^2-y^2=z$ 試求其交線上任一點之曲率與扭率。
2. 試求曲線 $(x-1)(y-2)=3$ 上任一點之曲率 κ 與扭率 τ。
3. 已知 $\kappa(s)=\dfrac{2}{5}$ 與 $\tau(s)=\dfrac{1}{5}$ 並且知道在點 $(2,0,0)$ 其 $\vec{T}(0)=(0,\dfrac{2}{\sqrt{5}},\dfrac{1}{\sqrt{5}})$、$\vec{N}(0)=(-1,0,0)$ 與 $\vec{B}(s)=(0,-\dfrac{1}{\sqrt{5}},\dfrac{2}{\sqrt{5}})$，試問該曲線表示式為何？
4. 試求空間曲面 $z=x^2y^2+y+2$ 在位置 $(1,0,2)$ 之單位法向量與曲面 $z=x^3y^3+x+3$ 在位置 $(0,0,3)$ 之單位法向量。
5. 求曲面 $x^2y+z=3$ 與 $x\ln z-y^2=-4$ 在交點 $(-1,2,1)$ 之夾角?
6. 試求曲面 $z=\sin(xy)$ 在位置 $(1,0,0)$ 之切平面與法線方程式。

參考解答:

1. 兩曲面交線可由 $x^2+y^2=1$ 可得 $x=\cos t$ 與 $y=\cos t$ 帶入 $x^2-y^2=z$ 可得 $z=\cos^2 t-\sin^2 t=\cos 2t$

$\therefore$ 兩曲面交線可由參數式表示，即 $\vec{r}(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+\cos 2t\,\hat{k}$

$$\vec{r}\,'(t)=-\sin t\,\hat{i}+\cos t\,\hat{j}-2\sin 2t\,\hat{k}$$

$$\left|\vec{r}\,'(t)\right|=(1+4\sin^2 2t)^{\frac{1}{2}}=(3-2\cos 4t)^{\frac{1}{2}}$$

$$\vec{t}(t)=\frac{\vec{r}\,'(t)}{\left|\vec{r}\,'(t)\right|}=\frac{-\sin t\,\hat{i}+\cos t\,\hat{j}-2\sin 2t\,\hat{k}}{(1+4\sin^2 2t)^{\frac{1}{2}}}$$

$$\vec{t}\,'(t)=-\frac{3\cos t-3\cos 3t+\cos 5t}{(1+4\sin^2 2t)^{\frac{3}{2}}}\hat{i}-\frac{3\sin t+3\sin 3t+\sin 5t}{(1+4\sin^2 2t)^{\frac{3}{2}}}\hat{j}-\frac{4\cos 2t}{(1+4\sin^2 2t)^{\frac{3}{2}}}\hat{k}$$

$$\left|\vec{t}\,'(t)\right|=\frac{\sqrt{11+6\cos 4t}}{1+4\sin^2 2t}$$

$$\kappa=\frac{\left|\vec{t}\,'(t)\right|}{\left|\vec{r}\,'(t)\right|}=\frac{\dfrac{\sqrt{11+6\cos 4t}}{1+4\sin^2 2t}}{(1+4\sin^2 2t)^{\frac{1}{2}}}=\frac{\sqrt{11+6\cos 4t}}{(1+4\sin^2 2t)^{\frac{3}{2}}}=\frac{\sqrt{5+12z^2}}{[1+4(1-z^2)]^{\frac{3}{2}}}$$

$$\vec{n}(t)=\frac{\vec{t}\,'(t)}{\left|\vec{t}\,'(t)\right|}=-\frac{3\cos t-3\cos 3t+\cos 5t}{(1+4\sin^2 2t)^{\frac{1}{2}}\sqrt{11+6\cos 4t}}\,\hat{i}-\frac{3\sin t+3\sin 3t+\sin 5t}{(1+4\sin^2 2t)^{\frac{1}{2}}\sqrt{11+6\cos 4t}}\,\hat{j}$$

$$-\frac{4\cos 2t}{(1+4\sin^2 2t)^{\frac{1}{2}}\sqrt{11+6\cos 4t}}\hat{k}$$

$$\vec{t}(t)\times\vec{n}(t)=\frac{-4\cos^3 t}{\sqrt{11+6\cos 4t}}\,\hat{i}+\frac{4\sin^3 t}{\sqrt{11+6\cos 4t}}\,\hat{j}+\frac{1}{\sqrt{11+6\cos 4t}}\hat{k}$$

$$\frac{d(\vec{t}(t)\times\vec{n}(t))}{dt}=\frac{-12\cos^2 t(11\sin t-5\sin 3t+\sin 5t)}{(11+6\cos 4t)^{\frac{3}{2}}}\,\hat{i}$$

$$+\frac{12\sin^2 t(11\cos t+5\cos 3t+\cos 5t)}{(11+6\cos 4t)^{\frac{3}{2}}}\,\hat{j}+\frac{12\sin 4t}{(11+6\cos 4t)^{\frac{3}{2}}}\hat{k}$$

$$\left|\frac{d(\vec{t}(t)\times\vec{n}(t))}{dt}\right|=\frac{6\sin 2t\sqrt{(1+4\sin^2 2t)}}{11+6\cos 4t}$$

$$\tau=\left|\frac{d\vec{B}(s)}{ds}\right|=\frac{d\left|\vec{b}(t)\right|}{ds}=\frac{dt}{ds}\frac{d\left|\vec{t}(t)\times\vec{n}(t)\right|}{dt}=\frac{1}{\left|\vec{r}\,'(t)\right|}\left|\frac{d(\vec{t}(t)\times\vec{n}(t))}{dt}\right|$$

$$=\frac{6\sin 2t}{11+6\cos 4t}=\frac{12xy}{5+12z^2}$$

另解：$\vec{r}\,'(t)=-\sin t\,\hat{i}+\cos t\,\hat{j}-2\sin 2t\,\hat{k}$

$$\vec{r}\,''(t)=-\cos t\,\hat{i}-\sin t\,\hat{j}-4\cos 2t\,\hat{k}$$

$$\vec{r}\,'''(t)=\sin t\,\hat{i}-\cos t\,\hat{j}+8\sin 2t\,\hat{k}$$

$$\vec{r}\,'(t)\cdot\vec{r}\,'(t)=1+4\sin^2 2t$$

$$\vec{r}\,'(t)\times\vec{r}\,''(t)$$

$$=(-4\cos t\cos 2t-2\sin t\sin 2t)\hat{i}+(2\sin 2t\cos t-4\sin t\cos 2t)\,\hat{j}+\hat{k}$$

$$=(-4\cos t\cos 2t-4\sin^2 t\cos t)\hat{i}+(4\sin t\cos^2 t-4\sin t\cos 2t)\,\hat{j}+\hat{k}$$

$$=[-4\cos t\cos 2t-2(1-\cos 2t)\cos t]\hat{i}+[2\sin t(1+\cos 2t)-4\sin t\cos 2t]\,\hat{j}+\hat{k}$$

$$=(-4\cos t\cos 2t-2\cos t+2\cos 2t\cos t)\,\hat{i}+(2\sin t+2\sin t\cos 2t-4\sin t\cos 2t)\,\hat{j}+\hat{k}$$

$$=(-2\cos t-2\cos t\cos 2t)\hat{i}+(2\sin t-2\sin t\cos 2t)\,\hat{j}+\hat{k}$$

$$\vec{r}\,'(t)\cdot[\vec{r}\,''(t)\times\vec{r}\,'''(t)]=\begin{vmatrix}-\sin t & \cos t & -2\sin 2t\\ -\cos t & -\sin t & -4\cos 2t\\ \sin t & -\cos t & 8\sin 2t\end{vmatrix}=6\sin 2t$$

$$\kappa=\frac{|\vec{r}'(t)\times\vec{r}''(t)|}{[\vec{r}'(t)\cdot\vec{r}'(t)]^{\frac{3}{2}}}=\frac{\sqrt{5+12\cos^2 2t}}{(1+4\sin^2 2t)^{\frac{3}{2}}}=\frac{\sqrt{5+12z^2}}{[1+4(1-z^2)]^{\frac{3}{2}}}$$

$$\tau=\frac{[\vec{r}'(t)\ \vec{r}''(t)\ \vec{r}'''(t)]}{|\vec{r}'(t)\times\vec{r}''(t)|^2}=\frac{6\sin 2t}{5+12\cos^2 2t}=\frac{12xy}{5+12z^2}$$

2. $(x-1)(y-2)=3 \quad \Rightarrow y=\frac{3}{(x-1)}+2$

$$\Rightarrow y'=\frac{-3}{(x-1)^2}$$

$$\Rightarrow y''=\frac{6}{(x-1)^3}$$

$$\kappa=\frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}}=\frac{\left|\frac{6}{(x-1)^3}\right|}{\left[1+\frac{9}{(x-1)^4}\right]^{\frac{3}{2}}}=\frac{6|(x-1)^3|}{[(x-1)^4+9]^{\frac{3}{2}}}$$

因為是平面曲線，所以$\tau=0$

3. 由 Frenet 定理可知:

$$\frac{d}{ds}\begin{Bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{Bmatrix}=\begin{bmatrix}0 & \kappa(s) & 0\\ -\kappa(s) & 0 & \tau(s)\\ 0 & -\tau(s) & 0\end{bmatrix}\begin{Bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{Bmatrix}$$

令 $P(s)=\begin{Bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{Bmatrix} \qquad \Rightarrow \vec{P}'(s)=A(s)\vec{P}(s)$

$$A=\begin{bmatrix}0 & \frac{2}{5} & 0\\ -\frac{2}{5} & 0 & \frac{1}{5}\\ 0 & -\frac{1}{5} & 0\end{bmatrix}$$

$$\det(A-\lambda I)=\begin{bmatrix}-\lambda & \frac{2}{5} & 0\\ -\frac{2}{5} & -\lambda & \frac{1}{5}\\ 0 & -\frac{1}{5} & -\lambda\end{bmatrix}=0 \quad \Rightarrow \lambda^3+\frac{\lambda}{5}=0 \quad \Rightarrow \lambda=0,\ \lambda=\pm\frac{i}{\sqrt{5}}$$

$$\lambda_1 = 0 \quad \Rightarrow v_1 = \begin{Bmatrix} 1 \\ 0 \\ 2 \end{Bmatrix}$$

$$\lambda_2 = i \quad \Rightarrow v_2 = \begin{Bmatrix} \frac{2}{\sqrt{5}} \\ i \\ -\frac{1}{\sqrt{5}} \end{Bmatrix}$$

$$\lambda_3 = -i \quad \Rightarrow v_3 = \begin{Bmatrix} -\frac{2}{\sqrt{5}} \\ i \\ \frac{1}{\sqrt{5}} \end{Bmatrix}$$

$$Q = \begin{bmatrix} 1 & \frac{2}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ 0 & i & i \\ 2 & -\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{i}{\sqrt{5}} & 0 \\ 0 & 0 & -\frac{i}{\sqrt{5}} \end{bmatrix}, \quad Q^{-1} = \begin{bmatrix} \frac{1}{5} & 0 & \frac{2}{5} \\ \frac{1}{\sqrt{5}} & -\frac{i}{2} & -\frac{1}{2\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & -\frac{i}{2} & \frac{1}{2\sqrt{5}} \end{bmatrix}$$

$$\vec{P}(s) = e^{As} \cdot \vec{P}(0)$$

$$= Qe^{Ds}Q^{-1} \begin{Bmatrix} 0\vec{i} + \frac{2}{\sqrt{5}}\vec{j} + \frac{1}{\sqrt{5}}\vec{k} \\ -\vec{i} + 0\vec{j} + 0\vec{k} \\ 0\vec{i} - \frac{1}{\sqrt{5}}\vec{j} + \frac{2}{\sqrt{5}}\vec{k} \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} + \frac{2}{5}(e^{-\frac{is}{\sqrt{5}}} + e^{\frac{is}{\sqrt{5}}}) & \frac{i}{\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}} - e^{\frac{is}{\sqrt{5}}}) & \frac{2}{5} - \frac{1}{5}(e^{-\frac{is}{\sqrt{5}}} + e^{\frac{is}{\sqrt{5}}}) \\ -\frac{i}{\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}} - e^{\frac{is}{\sqrt{5}}}) & \frac{1}{2}(e^{-\frac{is}{\sqrt{5}}} + e^{\frac{is}{\sqrt{5}}}) & \frac{i}{2\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}} - e^{\frac{is}{\sqrt{5}}}) \\ \frac{2}{5} - \frac{1}{5}(e^{-\frac{is}{\sqrt{5}}} + e^{\frac{is}{\sqrt{5}}}) & -\frac{i}{2\sqrt{5}}(e^{-\frac{is}{\sqrt{5}}} - e^{\frac{is}{\sqrt{5}}}) & \frac{4}{5} + \frac{1}{10}(e^{-\frac{is}{\sqrt{5}}} + e^{\frac{is}{\sqrt{5}}}) \end{bmatrix} \begin{Bmatrix} \frac{1}{\sqrt{5}}(2\vec{j} + \vec{k}) \\ -\vec{i} \\ -\frac{1}{\sqrt{5}}(\vec{j} - 2\vec{k}) \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} + \frac{4}{5}\cos(\frac{s}{\sqrt{5}}) & \frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) & \frac{2}{5} - \frac{2}{5}\cos(\frac{s}{\sqrt{5}}) \\ -\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) & \cos(\frac{s}{\sqrt{5}}) & \frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) \\ \frac{2}{5} - \frac{2}{5}\cos(\frac{s}{\sqrt{5}}) & -\frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}}) & \frac{4}{5} + \frac{1}{5}\cos(\frac{s}{\sqrt{5}}) \end{bmatrix} \begin{Bmatrix} \frac{1}{\sqrt{5}}(2\vec{j} + \vec{k}) \\ -\vec{i} \\ -\frac{1}{\sqrt{5}}(\vec{j} - 2\vec{k}) \end{Bmatrix}$$

$$=\begin{Bmatrix} \frac{1}{5\sqrt{5}}[1+4\cos(\frac{s}{\sqrt{5}})]\cdot(2\vec{j}+\vec{k})-\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\cdot(\vec{i})-\frac{2}{5\sqrt{5}}[1-\cos(\frac{s}{\sqrt{5}})]\cdot(\vec{j}-2\vec{k}) \\ -\frac{2}{5}\sin(\frac{s}{\sqrt{5}})\cdot(2\vec{j}+\vec{k})-\cos(s)\cdot(\vec{i})-\frac{1}{5}\sin(\frac{s}{\sqrt{5}})\cdot(\vec{j}-2\vec{k}) \\ \frac{2}{5\sqrt{5}}[1-\cos(\frac{s}{\sqrt{5}})]\cdot(2\vec{j}+\vec{k})+\frac{1}{\sqrt{5}}\sin(s)\cdot(\vec{i})-\frac{1}{5\sqrt{5}}[4+\cos(\frac{s}{\sqrt{5}})]\cdot(\vec{j}-2\vec{k}) \end{Bmatrix}$$

$$=\begin{Bmatrix} -\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}+\frac{2}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{1}{\sqrt{5}}\vec{k} \\ -\cos(\frac{s}{\sqrt{5}})\,\vec{i}-\sin(\frac{s}{\sqrt{5}})\,\vec{j} \\ \frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}-\frac{1}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{2}{\sqrt{5}}\vec{k} \end{Bmatrix}$$

所以可得

$$\vec{T}(s)=-\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}+\frac{2}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{1}{\sqrt{5}}\vec{k}$$

$$\vec{N}(s)=-\cos(\frac{s}{\sqrt{5}})\,\vec{i}-\sin(\frac{s}{\sqrt{5}})\,\vec{j}$$

$$\vec{B}(s)=\frac{1}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}-\frac{1}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{2}{\sqrt{5}}\vec{k}$$

由於 $\frac{d\vec{R}(s)}{ds}=\vec{T}(s)=-\frac{2}{\sqrt{5}}\sin(\frac{s}{\sqrt{5}})\,\vec{i}+\frac{2}{\sqrt{5}}\cos(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{1}{\sqrt{5}}\vec{k}$

$$\Rightarrow \vec{R}(s)=[2\cos(\frac{s}{\sqrt{5}})+c_1]\vec{i}+[2\sin(\frac{s}{\sqrt{5}})+c_2]\,\vec{j}+[\frac{s}{\sqrt{5}}+c_3]\vec{k}$$

又 $X(0)=2$，$Y(0)=0$ 與 $Z(0)=0$

$$X(s)=2\cos(\frac{s}{\sqrt{5}})+c_1,\quad X(0)=2 \quad \Rightarrow c_1=0$$

$$Y(s)=2\sin(\frac{s}{\sqrt{5}})+c_2,\quad Y(0)=0 \quad \Rightarrow c_2=0$$

$$Z(s)=\frac{s}{\sqrt{5}}+c_3,\quad Z(0)=0 \quad \Rightarrow c_3=0$$

$$\therefore\ \vec{R}(s)=2\cos(\frac{s}{\sqrt{5}})\vec{i}+2\sin(\frac{s}{\sqrt{5}})\,\vec{j}+\frac{s}{\sqrt{5}}\vec{k}$$

4. $z=x^2y^2+y+2$

$$\therefore\ \vec{r}=x\vec{i}+y\,\vec{j}+z\,\vec{k}=x\vec{i}+y\,\vec{j}+(x^2y^2+y+2)\vec{k}$$

在位置 $(1, 0, 2)$

$$\vec{r}_x=x\vec{i}+(2xy^2)\vec{k}=\vec{i}$$

$$\vec{r}_y=\vec{j}+(2x^2y+1)\vec{k}=\vec{j}+\vec{k}$$

$$\vec{n}=\frac{\vec{r}_x\times\vec{r}_y}{\left|\vec{r}_x\times\vec{r}_y\right|}=\frac{\vec{k}-\vec{j}}{\sqrt{2}}$$

$z=x^3y^3+x+3$

$\therefore\quad \vec{r}=x\vec{i}+y\vec{j}+z\vec{k}=x\vec{i}+y\vec{j}+(x^3y^3+x+3)\vec{k}$

在位置 $(0, 0, 3)$

$\vec{r}_x=\vec{i}+(3x^2y^3+1)\vec{k}=\vec{i}+\vec{k}$

$\vec{r}_y=\vec{j}+(3x^3y^2)\vec{k}=\vec{j}$

$$\vec{n}=\frac{\vec{r}_x\times\vec{r}_y}{\left|\vec{r}_x\times\vec{r}_y\right|}=\frac{\vec{k}-\vec{i}}{\sqrt{2}}$$

5. $x^2y+z=3 \quad \Rightarrow z=3-x^2y$

$\therefore\quad \vec{r}=x\vec{i}+y\vec{j}+(3-x^2y)\vec{k}$

曲面 $x^2y+z=3$ 在位置 $(-1, 2, 1)$ 之單位法向量為

$\vec{r}_x=1\vec{i}+0\vec{j}-2xy\vec{k}=\vec{i}+4\vec{k}$

$\vec{r}_y=0\vec{i}+1\vec{j}-x^2\vec{k}=\vec{j}-\vec{k}$

$$\vec{n}_1=\frac{\vec{r}_x\times\vec{r}_y}{\left|\vec{r}_x\times\vec{r}_y\right|}=\frac{-4\vec{i}+\vec{j}+\vec{k}}{\sqrt{18}}$$

$x\ln z-y^2=-4 \quad \Rightarrow z=e^{\frac{y^2-4}{x}}$

$\therefore\quad \vec{r}=x\vec{i}+y\vec{j}+z\vec{k}=x\vec{i}+y\vec{j}+e^{\frac{y^2-4}{x}}\vec{k}$

曲面 $x\ln z-y^2=-4$ 在位置 $(-1, 2, 1)$ 之單位法向量為

$$\vec{r}_x=1\vec{i}+0\vec{j}-e^{\frac{y^2-4}{x}}\cdot\frac{y^2-4}{x^2}\vec{k}=\vec{i}$$

$$\vec{r}_y=0\vec{i}+1\vec{j}+e^{\frac{y^2-4}{x}}\cdot\frac{2y}{x}\vec{k}=\vec{j}-4\vec{k}$$

$$\vec{n}_2=\frac{\vec{r}_\theta\times\vec{r}_z}{\left|\vec{r}_\theta\times\vec{r}_z\right|}=\frac{4\vec{j}+\vec{k}}{\sqrt{17}}$$

$\therefore$ 兩曲面之夾角為 $\vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1|\,|\vec{n}_2| \cos\theta$

$$\Rightarrow \theta = \cos^{-1}\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|} = \cos^{-1}\frac{5}{\sqrt{18}\,\sqrt{17}} = 1.2809 \ \text{(rad)} = 73.39^\circ$$

6. 由曲面 $z = \sin(xy)$ 可知 $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} = x\vec{i} + y\vec{j} + \sin(xy)\vec{k}$

在位置(1, 0, 0)

$$\vec{r}_x = 1\vec{i} + 0\vec{j} + y\cos(xy)\vec{k} = \vec{i}$$

$$\vec{r}_y = 0\vec{i} + 1\vec{j} + x\cos(xy)\vec{k} = \vec{j} + \vec{k}$$

$$\vec{n} = \frac{\vec{r}_x \times \vec{r}_y}{|\vec{r}_x \times \vec{r}_y|} = -\frac{1}{\sqrt{2}}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$$

切平面方程式為 $(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0 \quad \Rightarrow (x-1, y, z) \cdot (0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$

$$\Rightarrow -\frac{1}{\sqrt{2}}y + \frac{1}{\sqrt{2}}z = 0$$

$$\Rightarrow y - z = 0$$

法線方程式為 $(\vec{r} - \vec{r}_0) \times \vec{n} = 0$

$$\Rightarrow (x-1, y, z) \times (0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$$

$$\Rightarrow \frac{1}{\sqrt{2}}(y+z)\vec{i} - \frac{1}{\sqrt{2}}(x-1)\vec{j} - \frac{1}{\sqrt{2}}(x-1)\vec{k} = 0$$

$$\Rightarrow x = 1 \ \text{ 且 } \ y + z = 0$$

或是 $(\vec{r} - \vec{r}_0) \times \vec{n} = 0$ 即 $(\vec{r} - \vec{r}_0) // \vec{n} = 0$

$$\Rightarrow \frac{x-1}{0} = \frac{y}{-\frac{1}{\sqrt{2}}} = \frac{z}{\frac{1}{\sqrt{2}}} = t$$

$$\Rightarrow x = 1, \quad y = -\frac{1}{\sqrt{2}}t, \quad z = \frac{1}{\sqrt{2}}t$$

由 $y = -\frac{1}{\sqrt{2}}t$ 與 $z = \frac{1}{\sqrt{2}}t$ 可得 $y + z = 2$