

系級：_____ 學號：_____ 姓名：_____

1. 試解：

(1) $(x+2)^2 y'' - (x+2)y' + y = 3x+4$

(2) $x(x-1)y'' + xy' - y = 0$

(3) $y'' + (\sin x)y' + (\cos x)y = 0$

(4) $x(x+1)y'' + (4x+1)y' + 2y = 2x+1$

2. 已知 $y(x) = e^x$ 為方程式 $xy'' + 2(1-x)y' + (x-2)y = 0$ 之解，試求此方程式之通解。

3. 試解：

(1) $y'' = 1 + (y')^2$

(2) $yy'' + 3(y')^2 = 0$

(3) $xyy'' + x(y')^2 + 2yy' = 0$

參考解答：

1. (1) $y(x) = c_1(x+2) + c_2(x+2)\ln(x+2) + \frac{3}{2}(x+2)[\ln(x+2)]^2 - 2$

(2) $y(x) = c_1(1 + x \ln \frac{x-1}{x}) + c_2x$

(3) $y(x) = c_1 e^{\cos x} \cdot \int e^{-\cos x} dx + c_2 e^{\cos x}$

(4) $y(x) = \frac{1}{(x+1)^2} [c_1(x + \ln x) + c_2] + \frac{1}{3}(x+1)$

2. $y(x) = c_1 e^x + c_2 \frac{1}{x} e^x$

3. (1) $y = -\ln|\cos(x+c_1)| + c_2$

(2) $\frac{1}{4}y^4 = c_1x + c_2$ 或 $y = c$

(3) $y = \frac{1}{\sqrt{x}} \cdot \frac{c_2}{\sqrt{2+c_1x}}$