

系級：\_\_\_\_\_ 學號：\_\_\_\_\_ 姓名：\_\_\_\_\_

1. 給一微分方程  $x^2y'' + 2xy' - 12y = 0$ , 已知一補解為  $y_1 = x^3$ , 試求另一補解  $y_2$
2. 給一微分方程  $y'' + 2y' + y = 0$ , 已知其解為  $y_1 = e^{-x}$  (重根), 試用已知一解求另一補解找出  $y_2 = ?$
3. 試解下述微分方程:
  - (1)  $y'' - 8y' - 48y = 0$
  - (2)  $y'' - \sqrt{12}y' + 3y = 0$
  - (3)  $y'' + 4y' + 29y = 0$
  - (4)  $y''' + y'' + 3y' - 5y = 0$
  - (5)  $y''' + 6y'' + 12y' + 8y = 0$

<參考解答>

1.  $y_2 = x^{-4}$
2.  $y_2 = xe^{-x}$
3. (1)  $y = c_1e^{12x} + c_2e^{-4x}$ 
  - (2)  $y = c_1e^{\sqrt{3}x} + c_2xe^{\sqrt{3}x}$
  - (3)  $y = e^{-2x}(c_1 \cos 5x + c_2 \sin 5x)$
  - (4)  $y = c_1e^x + e^{-x}(c_2 \cos 2x + c_3 \sin 2x)$
  - (5)  $y = c_1e^{-2x} + c_2xe^{-2x} + c_3x^2e^{-2x}$