國立臺灣海洋大學河海工程學系 2019 工程數學(一) 2B 班第四次作業參考解答

系級：_____ 學號：_____ 姓名：_____

1. 試解下述 Riccati 微分方程:
   (1) $y' = e^{-x}y^2 - 3y + 3e^x$
   (2) $xy' + 3xy = y^2 + 2x^2 + y$

2. 試以全微分法求解下述微分方程
   (1) $(2y^2 - 6xy)dx + (3xy - 4x^2)dy = 0$
   (2) $(x - y^3)dy = ydx$
   (3) $(e^x \sin y + 3y)dx + (3x + e^x \cos y)dy = 0$
   (4) $2xydx + (1 + x^2)dy = 0$
   (5) $(2y + e^y + 6x^2)\dfrac{dy}{dx} + 4 + 12xy = 0$

3. 試以 Picard 法求解:
   $y' = 1 + y^2$, $y(0) = 0$

參考解答:

1. (1) $y = e^x + \dfrac{1}{\dfrac{e^{-x}}{2} + ce^x}$

   (2) $y = x + \dfrac{x}{1 + ce^x}$

2. (1) $x^2y^3 - 2x^3y^2 = c$
   (2) $\dfrac{x}{y} = -\dfrac{1}{2}y^2 + c$
   (3) $e^x \sin y + 3xy = c$
   (4) $x^2y = -y + c$
   (5) $y^2 + e^y + 4x + 6x^2y = c$

3. $y = \tan x$