國立臺灣海洋大學河海工程學系 2018 工程數學(一) 第一次隨堂測驗參考解答

系級：_____ 學號：_____ 姓名：_____

1. 試以變數變換法求解 $(3y+2x+4)dx-(4x+6y+5)dy=0$

$$(3y+2x+4)dx-(4x+6y+5)dy=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x+3y+4}{4x+6y+5}$$

令 $u=2x+3y \quad \Rightarrow \frac{du}{dx}=2+3\frac{dy}{dx} \quad \Rightarrow \frac{dy}{dx}=\frac{1}{3}\frac{du}{dx}-\frac{2}{3}$

$$\therefore \frac{dy}{dx}=\frac{2x+3y+4}{4x+6y+5} \quad \Rightarrow \frac{1}{3}\frac{du}{dx}-\frac{2}{3}=\frac{u+4}{2u+5}$$

$$\Rightarrow \frac{du}{dx}=\frac{7u+22}{2u+5}$$

$$\Rightarrow \int \frac{2u+5}{7u+22}du=\int dx$$

$$\Rightarrow \frac{2}{7}u-\frac{9}{49}\ln|7u+22|=x+C_1$$

$$\Rightarrow 7(x-2y)+3\ln|14x+21y+22|=C$$

2. 試以正合法求解 $\cos(x+y)dx+(3y^2+2y+\cos(x+y))dy=0$

$$\cos(x+y)dx+(3y^2+2y+\cos(x+y))dy=0$$

令 $M=\cos(x+y) \quad \Rightarrow \frac{\partial M}{\partial y}=-\sin(x+y)$

$N=3y^2+2y+\cos(x+y) \quad \Rightarrow \frac{\partial N}{\partial x}=-\sin(x+y)$

$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\therefore$ 此為正合 ODE

可知 $M=\frac{\partial \phi}{\partial x}=\cos(x+y) \quad \Rightarrow \phi=\sin(x+y)+f(y)$

$N=\frac{\partial \phi}{\partial y}=3y^2+2y+\cos(x+y) \quad \Rightarrow \phi=y^3+y^2+\sin(x+y)+g(x)$

比較兩式可得 $\phi(x,y)=\sin(x+y)+y^3+y^2=C$