國立臺灣海洋大學河海工程學系 2018 工程數學(一) 2A 班第一次作業參考解答

系級：_____　　學號：_____　　姓名：_____

試求下述微分方程式:

1. (1) $y' = \sec^2 x$

   (2) $y' = e^{2x-1} y^2$

2. $xy' = y^2 + y$ (hint: $u = \dfrac{y}{x}$)

3. $y' = 1 + 4y^2$, $y(1) = 0$

4. $4y'y - e^{x-y^2} = 0$, $y(1) = 2$

5. $y' = \dfrac{x^2 + xy + y^2}{x^2}$

參考解答:

1. (1) $y' = \sec^2 y \quad \Rightarrow \dfrac{dy}{dx} = \sec^2 y = \dfrac{1}{\cos^2 y}$

$$\Rightarrow \cos^2 y dy = dx$$

(兩邊積分) $\quad \Rightarrow \int \cos^2 y dy = \int dx$

$$\Rightarrow \int \frac{1+\cos 2y}{2} dy = x + c_1$$

$$\Rightarrow \frac{1}{2} y + \frac{1}{4} \sin 2y + c_2 = x + c_1$$

$$\Rightarrow \frac{1}{2} y + \frac{1}{4} \sin 2y = x + c$$

(2) $y' = e^{2x-1} y^2 \quad \Rightarrow \dfrac{dy}{dx} = e^{2x-1} y^2$

$$\Rightarrow \frac{1}{y^2} dy = e^{2x-1} dx$$

(兩邊積分) $\quad \Rightarrow \int \dfrac{1}{y^2} dy = \int e^{2x-1} dx$

$$\Rightarrow -\frac{1}{y} = \frac{1}{2} e^{2x-1} + c$$

2. $xy' = y^2 + y \quad \Rightarrow y' = \dfrac{y^2}{x} + \dfrac{y}{x}$

令 $u = \dfrac{y}{x} \quad \Rightarrow y = ux \quad \Rightarrow y' = u'x + u$

$$\therefore \quad y' = \frac{y^2}{x} + \frac{y}{x} \quad \Rightarrow u'x + u = u^2 x + u$$

$$\Rightarrow u' = u^2$$

$$\Rightarrow \frac{1}{u^2} du = dx$$

(兩邊積分) $\Rightarrow \int \frac{1}{u^2} du = \int dx$

$$\Rightarrow -\frac{1}{u} = x + c$$

$$\Rightarrow -\frac{x}{y} = x + c$$

$$\Rightarrow y = \frac{-x}{x+c}$$

3. $y' = 1 + 4y^2 \quad \Rightarrow \frac{1}{1+4y^2} dy = dx$

(兩邊積分) $\Rightarrow \int \frac{1}{1+4y^2} dy = \int dx$

令 $y = \frac{1}{2}\tan\theta \quad \Rightarrow y = \frac{1}{2}\sec^2\theta\, d\theta$

$\int \frac{1}{1+4y^2} dy = \int dx \quad \Rightarrow \frac{1}{2}\int \frac{\sec^2\theta}{1+\tan^2\theta} d\theta = \int dx$

$$\Rightarrow \int d\theta = 2\int dx$$

$$\Rightarrow \theta = 2x + c$$

$$\Rightarrow \tan^{-1}(2y) = 2x + c$$

$$\Rightarrow 2y = \tan(2x + c)$$

$$\Rightarrow y = \frac{1}{2}\tan(2x + c)$$

又 $y(1) = 0$ 此表示 $x = 1$ 時，$y = 0$ 帶回上式可得

$\tan(2 + c) = 0 \quad \Rightarrow 2 + c = 0$

$$\Rightarrow c = -2$$

$$\therefore \quad y = \frac{1}{2}\tan(2x - 2)$$

4. $2e^{y^2} = e^x + 2e^4 - e$

5. $\tan^{-1}\frac{y}{x} = \ln x + c$