國立臺灣海洋大學河海工程學系 2017 工程數學(一) 第一次隨堂測驗參考解答

系級：_____ 學號：_____ 姓名：_____

1. 試以分離變數法求解 $e^x y' = y\cos x$

$$e^x y' = y\cos x \quad \Rightarrow \frac{1}{y}dy = e^{-x}\cos x\,dx$$

$$\Rightarrow \int \frac{1}{y}dy = \int e^{-x}\cos x\,dx$$

$$\Rightarrow \ln|y| = \frac{1}{2}e^{-x}(-\cos x + \sin x) + c$$

2. 試以變數變換求解 $xy' = y^2 + y$ (hint: $u = \frac{y}{x}$)

$$xy' = y^2 + y \quad \Rightarrow y' = \frac{y^2}{x} + \frac{y}{x}$$

令 $u = \frac{y}{x} \quad \Rightarrow y = ux \quad \Rightarrow y' = u'x + u$

$$\therefore \quad u'x + u = u^2 x + u \quad \Rightarrow \frac{1}{u^2}du = dx$$

$$\Rightarrow \int \frac{1}{u^2}du = \int dx$$

$$\Rightarrow -\frac{1}{u} = x + c$$

$$\Rightarrow -\frac{x}{y} = x + c$$

$$\Rightarrow y = -\frac{x}{x+c}$$

另解:

$$xy' = y^2 + y \quad \Rightarrow \frac{1}{y^2+y}dy = \frac{1}{x}dx \quad \Rightarrow \int \frac{1}{y^2+y}du = \int \frac{1}{x}dx$$

$$\Rightarrow \ln|y| - \ln|y+1| = \ln|x| + \ln|c_1|$$

$$\Rightarrow \frac{y}{y+1} = c_1 x$$

$$\Rightarrow y = \frac{c_1 x}{1 - c_1 x}$$

$$\Rightarrow y = -\frac{x}{c+x}$$