

系級：_____ 學號：_____ 姓名：_____

1. 試求 $y'' - y = e^x$

$$\text{令 } y = e^{\lambda x} \Rightarrow (\lambda^2 - 1)e^{\lambda x} = 0 \Rightarrow \lambda = 1 \text{ or } -1$$

$$\therefore y_h = c_1 e^x + c_2 e^{-x}$$

$$\text{由待定係數法，令 } y_p = A \cdot x e^x \Rightarrow y'_p = A \cdot (x+1)e^x$$

$$\Rightarrow y''_p = A \cdot (x+2)e^x \text{ 代入 ODE 可得}$$

$$A \cdot (x+2)e^x - A \cdot x e^x = e^x \Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}$$

$$\therefore y_p = \frac{1}{2} x e^x$$

$$y = y_h + y_p = c_1 e^x + c_2 e^{-x} + \frac{1}{2} x e^x$$

2. $x^2 y'' + 6xy' + 6y = \ln x$

$$\text{令 } y = x^m \Rightarrow [m(m-1) + 6m + 6]x^m = 0 \Rightarrow m^2 + 5m + 6 = 0$$

$$\Rightarrow m = -2 \text{ or } -3$$

$$\therefore y_h = c_1 x^{-2} + c_2 x^{-3}$$

$$\text{由參數變變異法，令 } y_p = u_1 x^{-2} + u_2 x^{-3}$$

$$u'_1 = \frac{\begin{vmatrix} 0 & x^{-3} \\ \frac{\ln x}{x^2} & -3x^{-4} \end{vmatrix}}{\begin{vmatrix} x^{-2} & x^{-3} \\ -2x^{-3} & -3x^{-4} \end{vmatrix}} = \frac{x^{-5} \ln x}{x^{-6}} = x \ln x \Rightarrow u_1 = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2$$

$$u'_2 = \frac{\begin{vmatrix} x^{-2} & 0 \\ -2x^{-3} & \frac{\ln x}{x^2} \end{vmatrix}}{\begin{vmatrix} x^{-2} & x^{-3} \\ -2x^{-3} & -3x^{-4} \end{vmatrix}} = -\frac{x^{-4} \ln x}{x^{-6}} = -x^2 \ln x \Rightarrow u_2 = -\frac{1}{3} x^3 \ln x + \frac{1}{9} x^3$$

$$\therefore y_p = \left(\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2\right)x^{-2} + \left(-\frac{1}{3}x^3 \ln x + \frac{1}{9}x^3\right)x^{-3} = \frac{1}{6} \ln x - \frac{5}{36}$$

$$y = y_h + y_p = c_1 x^{-2} + c_2 x^{-3} + \frac{1}{6} \ln x - \frac{5}{36}$$