國立臺灣海洋大學河海工程學系 2016 工程數學(一) 第三次隨堂測驗參考解答

系級：＿＿＿＿＿＿　　學號：＿＿＿＿＿＿　　姓名：＿＿＿＿＿＿

1. 試以正合法求解 $(e^x + y)dx - dy = 0$

令 $M = e^x + y \quad \Rightarrow \dfrac{\partial M}{\partial y} = 1$

$N = -1 \quad \Rightarrow \dfrac{\partial N}{\partial x} = 0$

$\because$ 判別式 $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$ 　　$\therefore$ 此為非正合 ODE

又 $\dfrac{1}{N}(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}) = -1 \quad \Rightarrow$ 可得積分因子 $\mu = e^{\int \frac{1}{N}(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})dx} = e^{-\int dx} = e^{-x}$

$\therefore$ ODE 同乘積分因子後可得 $(1 + e^{-x}y)dx - e^{-x}dy = 0$

令 $M = \dfrac{\partial \phi}{\partial x} = 1 + e^{-x}y \quad \Rightarrow \phi = x - e^{-x}y + f(y)$

$N = \dfrac{\partial \phi}{\partial y} = -e^{-x} \quad \Rightarrow \phi = -e^{-x}y + g(x)$

$\therefore \ \phi(x,y) = x - e^{-x}y = c$

2. 試解此 Bernoulli ODE: $y' + \dfrac{1}{x}y = xy^4$

$y' + \dfrac{1}{x}y = xy^4 \quad \Rightarrow y^{-4}y' + \dfrac{1}{x}y^{-3} = x$

令 $u = y^{-3} \quad \Rightarrow u' = -3y^{-4}y'$

$\therefore \ y^{-4}y' + \dfrac{1}{x}y^{-3} = x \quad \Rightarrow -\dfrac{1}{3}u' + \dfrac{1}{x}u = x \Rightarrow u' - \dfrac{3}{x}u = -3x$ (此為一階線性 ODE)

積分因子 $\mu = e^{\int p(x)dx} = e^{-\int \frac{3}{x}dx} = x^{-3}$

同乘積分因子: $x^{-3}u' - 3x^{-4}u = -3x^{-3} \quad \Rightarrow \dfrac{d}{dx}(x^{-3}u) = -3x^{-2}$

$\Rightarrow \int d(x^{-3}u) = -\int 3x^{-2}\, dx$

$\Rightarrow x^{-3}u = 3x^{-1} + c$

$\Rightarrow y = (3x^2 + cx^3)^{-\frac{1}{3}}$