

系級：_____ 學號：_____ 姓名：_____

1. 試解: $y' = \frac{3y^2 - 2xy}{x^2}$ (hint: $u = \frac{y}{x}$)

$$y' = \frac{3y^2 - 2xy}{x^2} \Rightarrow y' = 3\left(\frac{y}{x}\right)^2 - 2\frac{y}{x}$$

$$\text{令 } u = \frac{y}{x} \Rightarrow y = ux \Rightarrow y' = u'x + u$$

$$\therefore y' = 3\left(\frac{y}{x}\right)^2 - 2\frac{y}{x} \Rightarrow u'x + u = 3u^2 - 2u$$

$$\Rightarrow \int \frac{1}{3u(u-1)} du = \int \frac{1}{x} dx$$

$$\Rightarrow \frac{1}{3} \int \left(\frac{1}{u-1} - \frac{1}{u} \right) du = \int \frac{1}{x} dx$$

$$\Rightarrow \frac{1}{3} (\ln|u-1| - \ln|u|) = \ln|x| + c_1$$

$$\Rightarrow \ln \left| \frac{y-x}{x^3 y} \right| = c_1$$

$$\Rightarrow y = \frac{x}{1 - cx^3}$$

2. 試以正合法求解 $12xy + 3x^2 + (6x^2 + 1)y' = 0$

$$12xy + 3x^2 + (6x^2 + 1)y' = 0 \Rightarrow (12xy + 3x^2)dx + (6x^2 + 1)dy = 0$$

$$\text{令 } M = 12xy + 3x^2 \Rightarrow \frac{\partial M}{\partial y} = 12x$$

$$N = 6x^2 + 1 \Rightarrow \frac{\partial N}{\partial x} = 12x$$

$$\therefore \text{判別式 } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

\therefore 此為正合 ODE

$$M = \frac{\partial \phi}{\partial x} = 12xy + 3x^2 \Rightarrow \phi = 6x^2 y + x^3 + f(y)$$

$$N = \frac{\partial \phi}{\partial y} = 6x^2 + 1 \Rightarrow \phi = 6x^2 y + y + g(x)$$

$$\therefore \phi(x, y) = 6x^2 y + x^3 + y = c$$