國立臺灣海洋大學河海工程學系 2016 工程數學(一) 第一次隨堂測驗參考解答

系級：_____ 學號：_____ 姓名：_____

1. 試解: $e^x y' = y\cos x$

$$e^x y' = y\cos x \quad \Rightarrow \frac{1}{y}dy = e^{-x}\cos x\,dx$$

$$\Rightarrow \int \frac{1}{y}dy = \int e^{-x}\cos x\,dx$$

$$\Rightarrow \ln|y| = \frac{1}{2}e^{-x}(-\cos x + \sin x) + C$$

2. 試解: $x^2 y' = y^2 + xy$

$$x^2 y' = y^2 + xy$$

$$\Rightarrow y' = \frac{y^2 + xy}{x^2} \longrightarrow \text{齊次型}$$

$$\Rightarrow y' = (\frac{y}{x})^2 + \frac{y}{x}$$

令 $u = \frac{y}{x} \quad \Rightarrow y = ux \quad \Rightarrow y' = u'x + u$

$$\therefore \; u'x + u = u^2 + u \quad \Rightarrow \frac{1}{u^2}du = \frac{1}{x}dx$$

$$\Rightarrow \int \frac{1}{u^2}du = \int \frac{1}{x}dx$$

$$\Rightarrow -\frac{1}{u} = \ln|x| + C$$

$$\Rightarrow -\frac{x}{y} = \ln|x| + C$$