

系級：_____ 學號：_____ 姓名：_____

1. (a) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x}$

(b) $\lim_{x \rightarrow \infty} \frac{\ln x}{e^x}$

2. 已知 $x^2 + 4y^2 = 4$

(a) 試求： $\frac{dy}{dx} = ?$

(b) 在點 $(\sqrt{2}, -\frac{1}{\sqrt{2}})$ 之斜率為何？

3. 下列何者為奇函數？何者為偶函數？

(a) $\cos 3x$ (b) $\sin 2x$ (c) $\cos 2x \sin 2x$ (d) $\sin 2x + \cos 2x$

4. (a) $\sin(\alpha + \beta) = ?$

(b) $\cos(\alpha + \beta) = ?$

5. 試求下列各函數 $f(x)$ 的微分 $f'(x) = ?$

(a) $f(x) = \sqrt{x^3 - 1}$

(b) $f(x) = \frac{2x}{x^3 + 1}$

(b) $f(x) = a^x$

6. (a) $\int \frac{1}{4x-1} dx = ?$

(b) $\int e^{2x} \cos 2x dx = ?$

(c) $\int_{-1}^1 |x| dx = ?$

7. $\begin{vmatrix} 3 & 3 & 3 \\ 2 & 2 & -2 \\ -1 & 1 & 1 \end{vmatrix} = ?$

8. $\begin{bmatrix} 1 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix} = ?$

9. $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ 請求出 A^{-1} 。

參考解答：

1. (a) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x} = 1$

(b) $\lim_{x \rightarrow \infty} \frac{\ln x}{e^x} = 0$

2. 已知 $x^2 + 4y^2 = 4$

(a) $\frac{dy}{dx} = -\frac{x}{4y}$

(b) $\frac{dy}{dx} = \frac{1}{2}$

3. 下列何者為奇函數？何者為偶函數？

(a) $\cos 3x$ (b) $\sin 2x$ (c) $\cos 2x \sin 2x$ (d) $\sin 2x + \cos 2x$

(a) 偶函數 (b) 奇函數 (c) 奇函數 (d) 不奇不偶

4. (a) $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$

(b) $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

5. (a) $f'(x) = \frac{d}{dx} \left[(x^3 - 1)^{\frac{1}{2}} \right] = \frac{1}{2} (x^3 - 1)^{-\frac{1}{2}} \cdot 3x^2 = \frac{3}{2} \frac{x^2}{\sqrt{x^3 - 1}}$

(b) $f'(x) = \frac{d}{dx} \left[\frac{2x}{x^3 + 1} \right] = \frac{2(x^3 + 1) - 2x \cdot 3x^2}{(x^3 + 1)^2} = \frac{-4x^3 + 2}{(x^3 + 1)^2}$

(c) $f'(x) = \frac{d}{dx} [a^x] = a^x \cdot \ln a$

6. (a) $\int \frac{1}{4x-1} dx = \frac{1}{4} \ln|4x-1| + C$

(b) $\int e^{2x} \cos 2x dx = \frac{1}{4} e^{2x} (\cos 2x + \sin 2x)$

(c) $\int_{-1}^1 |x| dx = \int_{-1}^0 -x dx + \int_0^1 x dx = 1$

7. $\begin{vmatrix} 3 & 3 & 3 \\ 2 & 2 & -2 \\ -1 & 1 & 1 \end{vmatrix} = 24$

8. $\begin{bmatrix} 1 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$

9. $A^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$