

系級：_____ 學號：_____ 姓名：_____

1. 試解下述微分方程：

(1) $y'' - 8y' - 48y = 0$

(2) $y'' - \sqrt{12}y' + 3y = 0$

(3) $y'' + 4y' + 29y = 0$

(4) $y''' + y'' + 3y' - 5y = 0$

(5) $y''' + 6y'' + 12y' + 8y = 0$

2. 請以待定係數法求解下述微分方程：

(1) $y'' - 4y' + 3y = e^{2x}$

(2) $y'' - 4y' + 3y = 2e^x$

(3) $y'' + 6y' + 9y = 12xe^{-3x}$

(4) $y'' + 4y' + 29y = 13\cos 5x$

(5) $y'' - 8y' - 48y = x$

3. 請以參數變異法求解下述微分方程：

$$y'' + 3y' + 2y = \frac{e^{-x}}{e^x + 1}$$

<參考解答>

1. (1) $y = c_1 e^{12x} + c_2 e^{-4x}$

(2) $y = c_1 e^{\sqrt{3}x} + c_2 x e^{\sqrt{3}x}$

(3) $y = e^{-2x}(c_1 \cos 5x + c_2 \sin 5x)$

(4) $y = c_1 e^x + e^{-x}(c_2 \cos 2x + c_3 \sin 2x)$

(5) $y = c_1 e^{-2x} + c_2 x e^{-2x} + c_3 x^2 e^{-2x}$

2. (1) $y = c_1 e^x + c_2 e^{3x} - e^{2x}$

(2) $y = c_1 e^x + c_2 e^{3x} - x e^x$

(3) $y = c_1 e^{-3x} + c_2 x e^{-3x} + 2x^3 e^{-3x}$

(4) $y = e^{-2x}(c_1 \cos 5x + c_2 \sin 5x) + \frac{1}{8} \cos 5x + \frac{5}{8} \sin 5x$

(5) $y = c_1 e^{12x} + c_2 e^{-4x} - \frac{1}{48}x + \frac{1}{288}$

3. $y = c_1 e^{-x} + c_2 e^{-2x} + x e^{-x} - (e^{-x} + e^{-2x}) \ln(e^x + 1)$