

系級：_____ 學號：_____ 姓名：_____

1. 試以正合微分方程法求解：

$$(1) 4x^4y^3 + 2y - x + (6x^5y^2 - x)y' = 0$$

$$(2) 1 + 2xye^y + x(xe^y - 1)y' = 0$$

$$(3) (xy^2 + 4xy^4)dx + (1 + x^2y + 4x^2y^3)dy = 0$$

2. 試以一階微分方程法求解：

$$(1) y' - \frac{1}{2}y = 2e^{\frac{5}{2}x}$$

$$(2) x^{-1}y' + x^{-2}y = x^{-3}$$

$$(3) \cos^2 x \cdot y' + y = \tan x$$

$$(4) \frac{dy}{dx} = \frac{y^2}{\cos y - 2xy}$$

3. 試解 Bernoulli ODE: $y' + \frac{1}{x}y = xy^4$

4. 試解: $y' = x(y-1)(y-2)$

<參考解答>

$$1. (1) 2x^2y^3 - \frac{y}{x^2} + \frac{1}{x} = c$$

$$(2) x^2y + xe^{-y} = c$$

$$(3) x^2y^2 + \tan^{-1} 2y = c$$

$$2. (1) y = e^{\frac{5}{2}x} + ce^{\frac{1}{2}x}$$

$$(2) xy = \ln|x| + c$$

$$(3) y = \tan x - 1 + ce^{-\tan x}$$

$$(4) y^2x = \sin y + c$$

$$3. y = (3x^2 + cx^3)^{\frac{1}{3}}$$

$$4. y = 2 + \frac{1}{ce^{\frac{x^2}{2}} - 1}$$