

系級：_____ 學號：_____ 姓名：_____

1. 下列何者為奇函數？何者為偶函數？

(a) $\sin 2x$ (b) $\cos 2x$ (c) $\cos x \sin x$ (d) $\sin x + \cos x$

2. (a) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$ (b) $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$

3. (a) $\sin(\alpha + \beta) = ?$ (b) $\cos(\alpha + \beta) = ?$

4. 試求：(a) $\frac{d}{dx}(\sin x \cos x)$ (b) $\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

5. 試求下列各函數的微分

(a) $f(x) = (x^2 - 1)^{\frac{2}{3}}$ (b) $f(x) = \ln(2x)$ (c) $f(x) = a^x$

6. (a) $\int \frac{1}{2x-2} dx = ?$ (b) $\int e^x \cos x dx = ?$

7. $z(x, y) = 2x \sin y - 3x^2 y^2$ ，試求全微分 $dz = ?$

8. $\begin{vmatrix} 2 & 4 & 6 \\ 1 & 1 & -1 \\ 0 & 1 & 5 \end{vmatrix} = ?$

9. $\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = ?$

10. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ 請求出 A^{-1} 。

參考解答：

1. 下列何者為奇函數？何者為偶函數？

(a) $\sin 2x$ (b) $\cos 2x$ (c) $\cos x \sin x$ (d) $\sin x + \cos x$

(a) 奇函數 (b) 偶函數 (c) 奇函數 (d) 均不是

$$2. (a) \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}[e^{2x} - 1]}{\frac{d}{dx}[x]} = \lim_{x \rightarrow 0} \frac{2e^{2x}}{1} = 2$$

$$(b) \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{\frac{d}{dx}[\ln x]}{\frac{d}{dx}[x]} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

3. (a) $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$

(b) $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$4. (a) \frac{d}{dx}(\sin x \cos x) = \cos x \cos x - \sin x \sin x = \cos 2x$$

$$\text{另解：} \frac{d}{dx}(\sin x \cos x) = \frac{d}{dx}\left(\frac{1}{2} \sin 2x\right) = \cos 2x$$

$$(b) \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cos x + \sin x \sin x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\text{另解：} \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{d}{dx}(\tan x) = \sec^2 x$$

$$5. (a) f'(x) = \frac{d}{dx}\left[(x^2 - 1)^{\frac{2}{3}}\right] = \frac{2}{3}(x^2 - 1)^{-\frac{1}{3}} \cdot 2x$$

$$(b) f'(x) = \frac{d}{dx}[\ln(2x)] = \frac{1}{x}$$

$$(c) f'(x) = \frac{d}{dx}[a^x] = a^x \cdot \ln a$$

$$6. (a) \int \frac{1}{2x-2} dx = \frac{1}{2} \ln|x-1| + c$$

$$(b) \int e^x \cos x dx = \frac{1}{2} e^x (\cos x + \sin x) + c$$

$$7. z(x, y) = 2x \sin y - 3x^2 y^2$$

$$\begin{aligned} \Rightarrow dz &= \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \\ &= (2 \sin y - 6xy^2) dx + (2x \cos y - 6x^2 y) dy \end{aligned}$$

$$8. \begin{vmatrix} 2 & 4 & 6 \\ 1 & 1 & -1 \\ 0 & 1 & 5 \end{vmatrix} = -2$$

$$9. \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 9 \\ -9 & 13 \end{bmatrix}$$

$$10. A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$