

系級：\_\_\_\_\_ 學號：\_\_\_\_\_ 姓名：\_\_\_\_\_

試以正合法求解下述微分方程：

1. (1)  $2xydx + x^2dy = 0$       (2)  $y' = e^{2x-1}y^2$

2.  $e^{x^2}(2xydx + dy) = 0$ ,  $y(0) = 2$

3. (1)  $2x \tan y dx + \sec^2 y dy = 0$

(2)  $(x^2 + y^2)dx - 2xydy = 0$

(3)  $e^{-y}dx + e^{-x}(-e^{-y} + 1)dy$ ,  $I = e^{x+y}$

試求解下述一階線性微分方程：

4.  $y' - 2y = -8x^2$

5.  $y' + \sec x \cdot y = \cos x$

6.  $y' - \frac{3}{x}y = 2x^2$

7.  $y' + \frac{2}{x+1}y = 3$ ,  $y(0) = 5$

試求解下述 Bernoulli 微分方程：

8.  $y' + xy = xy^2$

9.  $x^3y' = x^2y - y^3$

**參考解答：**

1. (1)  $x^2y = c$       (2)  $\frac{1}{2}e^{2x-1} + \frac{1}{y} = c$

2.  $y = 2e^{-x^2}$

3. (1)  $e^{x^2} \tan y = c$       (2)  $y = \sqrt{x^2 + cx}$       (3)  $e^x - y + e^y = c$ ,

4.  $y = 4x^2 + 4x + 2 + ce^{2x}$

5.  $y = \frac{x \cos x - \cos^2 x + c \cos x}{1 + \sin x}$

6.  $y = cx^3 + 2x^3 \ln|x|$

7.  $y = x + 1 + \frac{4}{(x+1)^2}$

8.  $y = \frac{1}{\frac{x^2}{1+ce^2}}$

9.  $\frac{1}{2} \frac{x^2}{y^2} = \ln|x| + c$